Air-Fuel Ratio Control of SI Engines with Individual Cylinder Fuel Decision: Periodic Time-Varying Model-Based Approach

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Abstract

As the vehicle population increases rapidly, reducing emissions is a growing challenge. In the vehicle with an internal combustion engine, the ratio of air mass to fuel mass, often called as air-fuel ratio, is an important index since it is closely related to emissions. Therefore, air-fuel ratio control is focused on in research institutes around the world. Fuel mass modification is widely used for air-fuel ratio control. With the development of electronic technology, fuel injection can be modified precisely and fuel injectors can be modified individually in multicylinder engines. Then, much higher performance air-fuel ratio control is required due to existing high performance electronic technology and requirement on emission reduction.

The difference or imbalance exists between cylinders in multicylinder engines and also our concerned air-fuel ratio is the one of mixed exhaust gases from multiple cylinders. Therefore, instead of previous air-fuel control which sends same fuel mass for all cylinders, fuel mass modification of individual cylinders are investigated for air-fuel ratio control.

This thesis focuses on air-fuel ratio control via fuel mass of individual cylinders. The fuel injection command of an individual cylinder is seen as the control input or manipulated variable, and the oxygen sensor output is seen as the controlled variable. Then the system with multiple cylinders is seen as a single cylinder system in which fuel injection command is for an individual cylinder at different time, and system parameters are time-varying due to cylinder-to-cylinder shift. Therefore, in this thesis, a multicylinder engine system is described as a single-input and single-output system and the control scale is
shortened to the operation interval between adjacent cylinders. That is the contribution of this thesis.

The modeling approach follows the physical process from fuel injection command to the oxygen sensor output and we obtain a linear periodic time-varying model. The periodicity is due to cylinder-to-cylinder shift. Also, a unified signal is introduced to represent fuel injection command of individual cylinders for a single input model. The modeling approach is investigated for both fuel injection cases including direct-injection case and port-injection case. Model parameters are identified using recursive least square estimation with a forgetting factor. Then the identified model is validated via experiments. Experiment results show the modeling approach is appropriate.

Learning control technology is used for learning unknown disturbances. The predictive control technology is also applied in air-fuel ratio control based on models. Considering practical requirements on the amplitude and the change rate of fuel injection, predictive control with input constraints is provided through solving an optimal problem with input constraints. Experiments are conducted for validating above three control strategies. Online learnt disturbances converge to fixed values and the air-fuel ratio under predictive control reaches the desired value, i.e., stoichiometric air-fuel ratio. With different constraint limits, the air-fuel ratio lines are shown in figures, and results show that the control performance is related to limits. Therefore, choosing appropriate constraint limits is useful for higher control performance.
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1

Introduction

1.1 Engine Control

Over the last three decades there has been a dramatic evolution in engine control systems in order to meet increased users’ requirement motivated by electrical and electronics engineering development. The new issues in engine control include improvement of fuel economy and drivability, and reducing emissions via dynamical control technologies. Fuel economy improvement requires improving the efficiency of combustion and energy management strategies. Drivability of a vehicle means the smooth delivery of power, better cold-start and warm-up performance, etc. Reducing emission is the study of reducing air pollutants for environment protection. A lot of successful control technologies have been developed by [1, 2, 3] which focus on one or multiple control issues.

Torque control is necessary for improving drivability and fuel economy. Torque is generated from the in-cylinder combustion which is influenced by air mass, fuel mass, sparking timing, etc. As actuating variable of torque control, the throttle angle, the variable cam timing, the spark advance, the injected fuel mass are usually used since they directly or indirectly influence the combustion [4, 5, 6, 7]. A predictive control method is proposed to track the torque reference in [5], in which the throttle angle is the control input. Specially, the proposed control method is based on a flow model where the mass flows through the throttle and the intake valves are regarded as the virtual control inputs. Actually, the behavior of the throttle is nonlinear due to its mechanical characteristics and the
significant friction forces [8]. Thus, the throttle model is often considered in the
torque control via throttle angle [8, 9]. In [8], a model is presented to characterize
the throttle body, while in [9] the throttle model is used for compensating its
actuation delay.

Ignition timing is often used for engine torque control. Briefly speaking, ig-
mination timing is the process of setting the angle relative to piston position and
crankshaft angular velocity that a spark will occur in the combustion chamber
near the end of the compression stroke. Adjusting ignition timing is to deliver
peak combustion pressures and improving the combustion performance. Various
closed-loop spark timing control schemes have been proposed based upon
in-cylinder pressure measurements [10, 11, 12] or spark ionization current sensing
[13, 14]. A spark timing controller is designed based on measurements of cylin-
der pressure and a parameter which evaluates the combustion in [10]. In other
words, the sparking timing is modified for in-cylinder combustion performance
and finally for torque control. Ignition timing is often modified for torque bal-
ancing in an internal combustion engine with multiple cylinders [15, 16, 17]. The
in-cylinder pressure at top dead center is used for evaluating the cycle-to-cycle
variability of generated torque in [16], and the spark advance is as the control
input for torque balancing. Moreover, valve cam timing (VCT) is also used for
torque control since VCT influences the air mass into cylinders [18].

Another important issue is reducing exhaust emissions in the research on en-
gine control. Exhaust emissions are pollutants which come out of a car’s tailpipe
when the engine is running. There emissions can reduce the quality of nearby
air [19, 20]. The effect can not be ignored when lots of cars are on the roads in
densely populated areas. Moreover, number of cars increase greatly, especially in
emerging economies like China, India, Brazil, etc. Many scientists believe that
the combined effect of exhaust emissions and other particulate released into the
atmosphere through industry is leading to global climate change [21, 22, 23, 24].
Therefore, a great number of researches focus on reducing exhaust emissions from
cars [25, 26].

A car engine produces a lot of emissions, some of which are treated inside the
car’s catalytic converter by a chemical process that make them less hazardous.
The catalytic converter in cars is a very important element of the car’s exhaust
1.1 Engine Control

system since it removes harmful nitrogen oxides ($NO_X$, $X = 1, 2, 3$) and carbon monoxide (CO) from the combustion residues before they are released into the environment. It is called a catalytic converter because it converts CO into ubiquitous $CO_2$ and $NO_X$ into $N_2$ and $O_2$ through chemical reactions on a solid catalyst [27, 28, 29]. Three-way catalyst converter (TWC) is generally used as the catalyst converter.

Figure 1.1 shows the conversion efficiency of TWC with air-fuel ratio, in which the TWC efficiency is about 98% when the air-fuel ratio is in a narrow region and drops abruptly outside the region [30, 31, 32]. Thus, for the TWC to operate efficiently, the air-fuel ratio is must be controlled at a desired value which is called as the stoichiometric value, about 14.7.

![Figure 1.1: TWC conversion efficiency with air-fuel ratio.](image)

In some cases, multiple control objectives are considered like [33, 34, 35] in which both engine torque and air-fuel ratio are as the control objectives. However, the control objectives are often interrelated or even conflicting. In the cases like cold-start and warm-up, the ratio of fuel mass to air mass is increased for high power [36, 37, 38, 39]. For example, cold-start problem requires engine start in much shorter time, and the air-fuel ratio is often ignored during the start-time process [36].
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1.2 Air-Fuel Ratio Control

Air-fuel ratio is the ratio of air mass to fuel mass present in an internal combustion engine. However, the concerned air-fuel ratio is the one of exhaust gas in the exhaust manifold before going into the catalyst converter. The oxygen sensor is generally mounted in the exhaust manifold for air-fuel ratio measurement [40, 41, 42]. Explicitly, it measures the proportion of oxygen in the gas flow and then air-fuel ratio is calculated from its sensed value. The sensor dynamics is often considered for high-performance air-fuel ratio control since current electrochemical devices such as universal exhaust gas oxygen (UEGO) sensors have a time constant of 100 ms or more [40, 43]. Also, in [40], a fast reacting device is developed based on a lambda sensor. The sensed air-fuel ratio is sent to the engine control unit (ECU) for fuel injection decision [44, 45].

Theoretically, air-fuel ratio can be modified through air charging or/and fuel injection. However, technically, fuel injection is often modified for air-fuel ratio control, since air charge is not so easily controlled while fuel injection can be modified slightly. That means the fuel injection is the control input in the air-fuel ratio control. The control output is the air-fuel ratio in exhaust manifold. Traditionally, fuel injection command is gotten from a map of speed, throttle angle, etc. in ECU. The method is safety for strong robustness [46, 47], while it is hard for high control performance of air-fuel ratio. Therefore, the closed-loop air-fuel ratio control is focused on. The closed-loop or feedback air-fuel ratio control means fuel injection command is modified with sensed air-fuel ratio as feedback as shown in Fig. 1.2. In ECU, the sensed air-fuel ratio is sampled and, based on the sampled value, fuel injection command is calculated for \( N \) cylinders in a \( N \)-cylinder system. And then sensor gets the air-fuel ratio of exhausted gases for next-step feedback control.

A great deal of control methods are applied into air-fuel ratio control. For example, the neural network technology is applied in air-fuel ratio modeling and control [39, 48, 49, 50]. The neural network model proposed in [50] uses information from multiple variables and considers engine dynamics to do multi-step ahead prediction, and the model is adapted in on-line mode to cope system uncertainty and time varying effects. Finally, the model predictive control technology
1.2 Air-Fuel Ratio Control

![Figure 1.2: Feedback air-fuel ratio control.](image)

is applied to engine air-fuel ratio control based on the neural network model. To improve this method, the adaptive control is combined with the learning algorithm for air-fuel ratio control in papers [51, 52], in which the adaptive technology and learning algorithm are used for on-line modification of model and control parameters.

In above control methods, same fuel injection command is issued for all cylinders in an engine with multiple cylinders. However, it is well known that difference exists among fuel injectors, air intake paths, cylinders, etc. That means same fuel injection command for all cylinders may lead to different air-fuel ratios of individual cylinders and further the perturbation of the air-fuel ratio in exhaust manifold.

Therefore, the individual air-fuel ratio control is proposed [42, 53]. The key of the control strategies is estimation of air-fuel ratios of individual cylinders from the sensed mixed air-fuel ratio. Usually, the estimation is obtained from a model that describes the behavior of gas mixing. A model is proposed in literature [54], which treats air-fuel ratios of individual cylinders and the sensed air-fuel ratio in manifold as the inputs and the output, respectively. Based on the model, a linear quadratic Gaussian (LQG) controller is presented and experimentally validated [39]. From a similar consideration, an inverse model with individual air-fuel ratios as outputs is presented [55, 56, 57]. Moreover, some papers focused on the diesel engines and proposed the approaches to estimate the individual air-fuel ratio by introducing an extended Kalman filter and nonlinear observer with the estimation of the exhaust manifold pressure [58, 59]. From the view of control, the individual air-fuel ratio control problem is a multiple-input and multiple-output (MIMO) system, i.e. the air-fuel ratio of the gas burnt in each cylinder is
1. INTRODUCTION

controlled via the fuel injection command of each cylinder, respectively, and as mentioned above, the key of this problem is to estimate the air-fuel ratio of each cylinder without extra sensors for each cylinder. Most of the literatures focused on the estimation of air-fuel ratio in each cylinder with single sensor installed at the gas mixing point. However, control of the air-fuel ratio in each cylinder is not crucial to control of the air-fuel ratio in exhaust gas at the gas mixing point, but, adjusting the fuel injection for each cylinder is necessary due to the cylinder-to-cylinder imbalance in the fuel paths. It makes the system multiple-input and single-output and causes complexity in system analysis and design.

Therefore, a single input signal, so-called unified injection command, is introduced in this paper to formulate the system as a single-input and single-output system (SISO). Another feature of the proposed model is periodic time-varying. Indeed, a periodic time-varying model has been used to represent the engine dynamics. For example, in the paper [56, 60], full-scaled engine model including intake manifolds, fuel injectors and rotational dynamics is formulated as periodic time-varying MIMO system. It should be noted that the periodicity in the model [60] results from the cylinder-to-cylinder combustion-event shifting. In contrast, the periodicity in the proposed model is caused by the restored parameter cyclically under the assumption that the exhaust gas remains in runners less than one cycle.

1.3 Contribution of This Dissertation

In this thesis, a modeling approach is provided for the air-fuel ratio of exhaust gas in an engine with multiple cylinders. The modeling follows the whole physical process from individual cylinder fuel injection to the air-fuel ratio sensor output. A linear periodic time-varying model is derived with BDC (bottom dead center) scale which is shorter than the cycle scale in previous research [61, 62]. Moreover, disturbances in fuel paths are considered in the modeling and the model is single-input and single-output [63]. The contribution of this modeling provides a basis for air-fuel ratio control via individual cylinder fuel injection.

Based on the model, learning control and predictive control technologies are applied into air-fuel ratio control. Iterative learning control is used for unknown
disturbances learning, and with on-line learning, fuel injection command is modified using predictive control [64]. Also, the input constraints are considered for air-fuel ratio control [65]. Experiments are conducted to validate above modeling approach and control strategies. The thesis layout is given by details as follows.

Chapter 2 Preliminaries: First of all, it is necessary to understand the whole physical process of an engines system. As preparation, this chapter introduces the process from fuel injection command to the air-fuel ratio sensor output. Fuel injection command is sent individually and serially for individual cylinders. Cylinders works serially and variability exists between cylinders. It wastes time when burnt gas passes runners. Mixing phenomenon in exhaust manifold and sensor dynamics are also introduced. The introduction in Chapter 2 makes it easy to understand the operation mechanism of a multicylinder engine and where the dynamics exists.

Chapter 3 Experiment Setup: Experiments are conducted on a same test bench. In this thesis, a separate chapter is devoted to introduce the experiment setup. The engine used is a V6 engine and the dynamometer is a low inertia one. Except the engine-dynamometer mechanism, this chapter introduces some main experimental tools like dSPACE, sensors, SPARC, etc. At the end of this chapter, experiment steps are given for experiment operation of identification, model validation, and validating control strategies.

Chapter 4 Air-Fuel Ratio Model: This chapter aims to propose the modeling approach for air-fuel ratio in an engine with multiple cylinders. First, the whole physical process is divided into multiple parts, and they are described as mathematical models using mathematical concepts and language. Second, the whole system is described as a linear time-varying model with single input and single output. The model input is the fuel injection command which represents command for different cylinders at different time. The model output is the sensor output, i.e., sensed air-fuel ratio. Third, the modeling approach is extended to the port-injection engine. The model is also linear time-varying, while its structure is much more complex than the model in direct injection case since the wall-wetting phenomenon is considered in port injection case. Finally, the modeling approach for both cases are validated by experiments on a V6 engine test bench which is introduced in Chapter 3.
Chapter 5 Model-Based Air-Fuel Ratio Control: This chapter aims to design control strategies for air-fuel ratio control based on models proposed in Chapter 4. For an individual fuel injector, the disturbance is considered as being fixed. In the air-fuel ratio model, a single variable is introduced which periodically represents the disturbances of all the cylinders, respectively. The learning technology is often used for periodic system control, and it is applied to the unknown periodic disturbance learning. The predictive control technology is used for air-fuel ratio control. The control strategy is gotten by solving the optimal problem of predicted control results. Also, the solution of the optimal problem is given when the input constraints are considered. Finally, above control strategies are validated by experiment.

Chapter 6 Conclusion: Some concluding remarks are given on this research in this chapter. Moreover, the list is given on the future work and the extension of this research.
2

Preliminaries

In the ignition-combustion engines, fuel is combusted with air in cylinders, and the products or burnt gases are exhausted outside. This thesis focuses on the air-fuel ratio of the burnt gases in the exhaust manifold before passing through the three-way catalytic converter (TWC). This chapter aims to introduce fundamentals of the engine system related to the contents of this thesis, i.e., from the fuel injection command to the air-fuel ratio sensor output. Moreover, a brief review of the individual cylinder air-fuel ratio control is given which motivates the idea of periodic description of the engine system.

2.1 System Structure of a Multicylinder Engine

This section mainly aims to introduce the system structure of multicylinder engines. In an ignition-combustion engine with multiple cylinders, an individual cylinder has its air intake path, fuel injector, and exhaust runner. Air intake paths share an intake manifold which is controlled by a throttle (only consider this kind of engine in this thesis). Fuel injection command is sent by the engine control unit (ECU). Multiple cylinders share an exhaust manifold and a TWC. The structure of an engine with three cylinders is schemed in Fig. 2.1. Fuel injection of three cylinders is excited by the engine control unit (ECU) cyclically. Then, the three cylinders are numbered as No. 1, No. 2, and No. 3 according to combustion sequence. In a four-stroke engine, an operating cycle requires two revolutions, i.e., 720° in the crankshaft angle. The operating interval between
two adjacent cylinders is $240^\circ$ or $2\pi/3$ [rad]. In each cylinder, the mixture of injected fuel and charged air is sparked and the burnt gas is exhausted outside into their corresponding runners. The action including combustion and burnt gas exhausting is also in a $2\pi/3$-interval sequence for three cylinders. Burnt gases from three cylinders pass through their runners and goes into the public exhaust manifold. Here, an oxygen sensor is mounted for air-fuel ratio measurement, and the sensed value is gotten by ECU.

![Diagram](image)

**Figure 2.1:** The exhaust system in multicylinder engines.

The whole process can be divided into five parts: fuel injection, in-cylinder combustion, passing through runners, mixing in exhaust manifold, and air-fuel ratio measurement. Fig. 2.2 provides a block diagram of the whole process in which each block represents a physical behavior. In Fig. 2.2, $u_{fi}$ represents the fuel injection command to the fuel injector of No. $i$ cylinder. $m_{ai}$ and $m_{fi}$ denote the charged air mass and the injected fuel mass into No. $i$ cylinder, respectively. For No. $i$ cylinder, the gas flow passing through the exhaust valve, and the air-fuel ratio of the gas flow are denoted by $\dot{m}_{ei}$ and $\eta_i$, respectively. At gas mixing point, gas flow of individual cylinders mixes, and so does individual air-fuel ratios. The oxygen sensor output is denoted by $\eta_s$.

From the view of dynamical system, the whole process is a multiple-input and single-output system, which is driven by the fuel injection command of cylinders, and the output is the sensed air-fuel ratio. In port injection case, fuel path
2.2 Individual Cylinder Fuel Injection

For direct injection, fuel injection of an individual cylinder is excited by ECU at a certain time before in-cylinder combustion. The delivery time of the command can be modified slightly on a range before sparking. Also, in the case of port injection, fuel injection is conducted during intake stroke. The engine ECU changes the fuel injection volume by changing the injection duration based on measured fuel pressure in fuel path. For multiple cylinders, fuel injection command is sent individually and serially with same interval of the crankshaft angle. During an engine cycle, fuel injection command for all cylinders is renewed once.

Fig. 2.3 shows a timetable of sending fuel injection command in a $N$-cylinder engine, in which open circles, solid circles, open squares, and solid squares represent fuel injection command to fuel injectors of No. 1, No. 2, No. $i$, and No. $N$ cylinders, respectively. The fuel injection can be seen as discrete-time event in a sequence from No. 1 cylinder to No. $N$ cylinder and again. As presented in 2.1, an engine cycle is $2\pi$ [rad] in crankshaft angle, thus the interval between adjacent sending is $2\pi/N$. Assuming the engine speed is fixed, the time interval between adjacent command sending is $T_c/N$ where $T_c$ represents an engine-cycle time.
2. PRELIMINARIES

Injected fuel mass is determined directly by injector open duration and fuel pressure in fuel path. Hence, fuel pressure fluctuates cause the variation of fuel injecting mass. Also, clogged fuel injector influences injection operation. We call the error above as the disturbance on a fuel injector. More specially, the disturbance is the error between the fuel injection command to a fuel injector and its real injected fuel mass. In direct-injection case, the injected fuel mass is the mass into a cylinder, and in port-injection case, it is just the injected fuel mass into air path. The existence of disturbances requires real-time fuel compensation for air-fuel ratio control. Furthermore, the difference on different injectors leads to the imbalance of injected fuel mass between cylinders. For high performance air-fuel ratio control, the imbalance can not be ignored.

However, disturbances are unknown and can not be measured directly. Since we can not directly measure the real injected fuel mass each cycle into a cylinder, even though fuel injection command is easily known. To illustrate the existence of disturbances, Fig. 2.4 shows air-fuel ratio of individual cylinders under same fuel injection command. The experiments are conducted on a three-cylinder engine. During the experiment, the engine works under fixed operation conditions including fixed throttle angle, fixed speed, and fixed load. Therefore, air charge to each cylinder can be considered as a constant. The top plot of Fig. 2.4 shows the same fuel injection command for three cylinders. In the bottom plot, three lines represent three cylinders’ air-fuel ratios, respectively. Obviously, the difference between cylinders is due to the disturbance.

In a port-injection gasoline engine the injected fuel is into the air stream in
2.2 Individual Cylinder Fuel Injection

![Graph showing fuel injection command vs time for individual cylinders](image)

**Figure 2.4:** Air-fuel ratios of individual cylinders under same fuel injection command

the form of a jet of liquid. Atomizing into small droplets, these mix with the air, begin to vaporize and eventually get carried off past the intake value and into the cylinder. During the process, varying amounts of this fuel might condense on the wall, travel along it and eventually vaporize back into the air stream as shown in Fig. 2.5. However, the presence of this fuel, often referred to as a puddle can have a significant effect on the air-fuel ratio eventually presented to the engine.

![Diagram of wall-wetting phenomenon](image)

**Figure 2.5:** Wall-wetting phenomenon in port-injection engine.

For further understanding port injection, an experiment is conducted to show fuel mass into a cylinder under direct injection and port injection, respectively.
2. PRELIMINARIES

Oxygen sensor is equipped at exhaust valve port for featuring fuel mass in the cylinder. Figure 2.6 provides the sensor output during fuel injection command changing. Obviously, the response time of charged fuel mass in port injection is much longer than the one of direct injection, and the experiment demonstrates wall-wetting phenomenon influences the fuel mass into cylinders obviously.

Figure 2.6: Sensed fuel-air ratio under direction and port injection, respectively.

2.3 Gas Mixing in Exhaust Manifold

In an exhaust system, exhaust gases from multiple cylinders pass through their runners and are collected in one pipe. Finally they go into the surrounding after converting in TWC. We call the collection in one pipe as gas mixing. For sake of illustration, define a mixing point in the public horizon of exhaust manifold. The process can be seen as gases from multiple cylinders mix together at the mixing point and then the mixture runs into TWC.
2.3 Gas Mixing in Exhaust Manifold

Fig. 2.7 shows a photo of an exhaust system, in which No. 1, No. 2, No. 3 cylinders share an exhaust manifold. As shown in Fig. 2.7, gases from cylinders have individual runners and different running length from an exhaust valve to the mixing point.

Exhaust valve of a cylinder opens during the exhaust stroke. Rather, the exhaust valve often opens a little before exhaust stroke and closes a little after exhaust stroke. In four-stroke engines, exhaust stroke is during $540^\circ$ to $720^\circ$ in crank angle with assigning $0^\circ$ to the beginning of intake stroke. For multiple cylinders, their exhaust valve open and close serially.

Fig.2.8 shows the exhaust-valve opening time in an exhaust system with three cylinders. Three valves are numbered as No. 1, No. 2, and No. 3 in operation sequence, and the operation interval is $240^\circ$. Thus, there is no overlap between two adjacent openings in this exhaust system.

The gas travels from the exhaust valve to the mixing point with dynamics, which can be treated as the first order dynamical system. At the exhaust valve of No. $i$ cylinder, the gas flow is $\dot{m}_{ei}$ and it is the input of the dynamical system. At the mixing point, gas flow from No. $i$ cylinder is $\dot{m}_{si}$ and it is the output of the dynamical system. Actually, the total gas flow at the mixing point is the sum of gas flow of all cylinders which share the exhaust manifold.

Fig. 2.9[a] shows gas flow at individual exhaust valve points, in which $\theta_{ci}$ ($i = 1, 2, 3$) denotes the exhaust valve closing time of No. $i$ exhaust valve. Fig. 2.9[b]
2. PRELIMINARIES

shows the gas flow at mixing point from individual cylinders, in which $\theta_{oi}$ ($i = 1, 2, 3$) denotes the exhaust valve opening time of No. $i$ exhaust valve.

Figure 2.8: Exhaust-valve opening time.

Figure 2.9: Gas flow at exhaust valve points and the mixing point.
2.4 Air-Fuel Ratio

Air-fuel ratio is the mass ratio air to fuel present in an internal combustion engine. Roughly speaking, it is seen as the ratio during combustion without consideration of residual gases. Thus, it is an important index for tuning performance of in-cylinder combustion. It is also important for anti-pollution since combustion products are decided by the ratio, and researches show that an appropriate ratio make less exhaust emissions. In the engines with multiple cylinders, air-fuel mixture is combusted in individual cylinders and cylinders have their individual air-fuel ratios. In this thesis, the air-fuel ratio of an individual cylinders is called as the individual cylinder air-fuel ratio, and call the air-fuel ratio of all cylinders be a mixed air-fuel ratio or air-fuel ratio.

An oxygen sensor or lambda sensor is mainly used for the measurement of air-fuel ratio, which measures the proportion of oxygen in the burnt gas. Then the air-fuel ratio is calculated from the proportion based on a general air density and fuel type. In other words, in one combustion event, the air-fuel mass ratio is gotten by measuring the oxygen proportion of its burnt gas.

The sensor has its dynamics response to air-fuel ratio change. Fig. 2.10 shows the sensed air-fuel ratio under fuel injection change. During the experiment, air charge is fixed, and fuel mass is changed (in direct-injection case).

Therefore, the real air-fuel ratio is considered as rising steeply. Slowly changing of sensed fuel-air ratio is due to sensor dynamic response. In later chapters of this thesis, sensor dynamics is considered in air-fuel ratio modeling.

Oxygen sensor is mounted near an exhaust valve for measuring an individual cylinder air-fuel ratio. When the exhaust valve opens and the in-cylinder burnt gas starts from the exhaust valve point. Actually it measures the real-time oxygen density of burnt gas flow, and the sensed value may rise and fall due to noise, measurement error, and specially, uneven molecular size and mixture. Even so, the sensed air-fuel ratio is as the individual cylinder air-fuel ratio.

Oxygen sensor is mounted at the mixing point in exhaust manifold for measuring the air-fuel ratio of the engine. The mounting position is before TWC, since the concerned air-fuel ratio is just the one of exhaust gas into TWC. In ex-
haust manifold, the gas flow is the addition of gas flows from multiple cylinders, and the air-fuel ratio is the mixture of individual cylinder air-fuel ratios.

Fig. 2.11 shows air-fuel ratios of individual cylinders and their mixed air-fuel ratio. Individual cylinder air-fuel ratios are measured in their corresponding exhaust valve points. The mixed air-fuel ratio is measured at the mixing point. Working conditions are fixed during the experiment. Sensed values show the imbalance between cylinders and the difference between individual cylinder and mixed air-fuel ratios.

2.5 Previous Work on Individual Cylinder Air-Fuel Ratio Control

Imbalance exist in fuel paths as presented in Section 2.2. Air charge into individual cylinders is different. Also, the length of individual cylinder runners is different as presented in Section 2.3. We call above imbalance and difference as the cylinder-to-cylinder imbalance in multicylinder engines. The existence of cylinder-to-cylinder imbalance requires individual fuel injection for high perfor-
2.5 Previous Work on Individual Cylinder Air-Fuel Ratio Control

Figure 2.11: Sensed air-fuel ratios at exhaust valve points and mixing point.

mance control of the air-fuel ratio. In this subsection, a brief review of air-fuel ratio control via individual fuel injection is presented which exactly is individual cylinder air-fuel ratio control.

Our concerned air-fuel ratio is the mixed air-fuel ratio and it is the mixture of individual cylinder air-fuel ratios as mentioned in Section 2.4. Therefore, when individual air-fuel ratios keep around the stoichiometric value, then the mixed air-fuel ratio is stabilized around the stoichiometric value. That is the idea of individual air-fuel ratio control. Explicitly, individual air-fuel ratio control is to guarantee individual air-fuel ratios around desired value via their corresponding fuel injection, and it is finally for controlling air-fuel ratio of exhaust gas in exhaust manifold.

Usually, there are no sensors for measuring individual cylinder air-fuel ratios due to the high cost. Therefore, an observer is designed to estimate individual cylinder air-fuel ratios. For example, in the literature [54], a model is gotten following the physical process of gas mixing in the exhaust manifold, and an input observer is designed. In the model, model inputs are individual cylinder air-fuel ratios, and their mixed air-fuel ratio is the model output.

Fig. 2.12 shows the control structure of the individual cylinder air-fuel ratio
2. PRELIMINARIES

control. Here are three cylinders sharing an exhaust manifold in the engine. The sensor is the one mounted in exhaust manifold and the sensor output is the sensed mixed air-fuel ratio. After being sampled, the sensed air-fuel ratio is used for estimating individual cylinder air-fuel ratios. Finally, the estimation is used to modify fuel injection of individual cylinders. Above operation including sampling, estimation, and control input decision is finished in the ECU. In this control, fuel injection of cylinders are modified individually to solve the cylinder-to-cylinder imbalance.

![Diagram](image_url)

**Figure 2.12:** Individual cylinder air-fuel ratio control.

Experiments are conducted on the test bench with a V6 engine to validate the control strategy. The test bench and experimental operation will be introduced in details in Chapter 3.

During the experiment, let the engine work under the rotational speed 1200[rpm] and a load 80[Nm] added on the engine. The sensor is the one mounted at the mixing point in exhaust manifold. The sensor output is sampled at exhaust BDCs. Use the inverse model in [56] to estimate air-fuel ratios of three cylinders. Also, we can get the model output, i.e., estimated mixed air-fuel ratio, with observed model inputs. Finally, fuel injection is modified with estimated individual cylinder air-fuel ratios as feedback.

Fig. 2.13 shows the experimental results. Fuel injection command for three cylinders is shown in the top plot. The top second, third, and fourth plots show estimated and measured air-fuel ratios of three cylinders, in which blue lines are estimation and red lines are measurement. The measured air-fuel ratio of mixed gas and its estimated values are shown in the bottom plot. At the beginning, same fuel injection command is sent for three cylinders. Estimator starts and
estimated individual cylinder air-fuel ratios can be gotten. Then the controllers
starts at about 7[ssecond]. Fuel injection command of three cylinders are different
due to their imbalance. Individual air-fuel ratios individually rise up or fall down
to the value about 14.7. At the mean time, air-fuel ratio of mixed gas also climbs
up to the stoichiometric value 14.7. Experimental results validate the control
strategy - individual cylinder air-fuel ratio control.

Furthermore, robustness of individual air-fuel ratio control is investigated via
experiments. The control performance is investigated when perturbations are
added on feedback or/and control inputs as shown in Fig. 2.14. Explicitly, mul-
tiply gains on fuel injection command and add sensor perturbations on sensed
signals based on original control system. A great deal of experimental results
are given in [47], and results show that control performance is closely related to
estimation precision.

Here, we have a point that air-fuel ratio control can be improved by modifying
individual fuel injection decision. Also, great number of researches focus on this
kind of research like individual air-fuel ratio control. For us, we will give a periodic
description and design control methods for air-fuel ratio control with individual
fuel injection decision.

2.6 Periodic Description of the Plant

Recall the multicylinder engine system from fuel injection command to the air-
fuel ratio of exhaust gas. Fuel injection command is sent for individual cylinders
in a sequence, and burnt gases of the cylinders are exhausted outside in a same
sequence. Each cylinder has its self fuel path, fuel injector, and exhaust runner.
Finally, we concern the air-fuel ratio of exhaust gas in public exhaust manifold.

The whole physical process can be seen as a system with fuel injection com-
mand as the input and sensed air-fuel ratio as the output. Command sending is
actually discrete-time event and the air-fuel ratio is continuous. Fig. 2.15 shows
the schematic of the system of a three-cylinder engine. The system is excited by
three inputs $u_f$, $u_f^2$, and $u_f^3$.

One of main features of the system is periodicity like periodic fuel injection
excitation, periodically updating exhaust gas in exhaust manifold, etc. Therefore,
Figure 2.13: Experiment results of individual air-fuel ratio control.
Figure 2.14: Air-fuel ratio control with individual fuel injection.

Figure 2.15: Cyclically and serially excited input signals.
the system is periodic and the periodicity is due to cylinder-to-cylinder shift. Also, the system is time-varying due to cylinder-to-cylinder variation.

Here is an idea that a multicylinder engine is seen as an engine with a single cylinder. The single cylinder is fictitious, and it is time-varying. The time-varying is due to that it acts different cylinders at different time. Due to similar reasons, the single cylinder is with time-varying fuel paths, time-varying air paths, and time-varying runner. Then, the multicylinder engine system is seen as a system with single input and single output. The single input is the fuel injection command for the engine and the single output is the air-fuel ratio of exhaust gas from the engine. For the engine shown in Fig. 2.15, Fig. 2.16 provides an equivalent engine with a single cylinder. Fuel injection command is issued to the single cylinder with BDC scale, and with same scale, burnt gas is exhausted outside.

![Figure 2.16: Equivalence to a SISO system.](image)

![Figure 2.17: Control structure of the periodic single-input system.](image)

The idea describes the multi-input system as a single-input system and that is useful for control design, and the scale is shortened to BDC scale from engine-cycle scale.
Following the idea, design a control for the SISO system. The system input is discrete-time event and the output is continuous time single. Therefore, for control design, the sensor output is sampled with BDC interval. Fig. 2.17 shows the control structure, in which the system output is sampled at BDCs. In practice, the control input switches to different channels for different fuel injectors.
Experiment Setup

This chapter mainly introduces the test bench on which experiments are conducted in this thesis. The whole test bench can be divided into the combination of engine and dynamometer and their accessories. Specifically, it consists of an engine, a dynamometer, sensors, meters, control system, and operation platform. It is necessary to introduce types of devices, their mechanism and connection. The control desk is introduced in the last section, including system communication, programming and software platform, and the experiment operation.

3.1 Test-Bench Facilities

All the experiments are conducted on the test bench, and the core of the test bench is a multicylinder engine with a dynamometer. We need an engine and a dynamometer for monitoring operation states of the engine. Electronic controllers are with amplifiers and actuators. For user communication, experiment software is needed for electronic control. Figure 3.1 provides a whole structure of the test bench.

The engine is an entire one, and it has its air intake, fuel injection system, exhaust system, lubricant oil system, and cooling system. Through clutch and transmission, a dynamometer is connected with the engine as shown in Figure 3.1 in which we use a manual transmission (MT). The dynamometer is for measuring torque produced by the engine, and it also monitors or drives the engine. The
3. EXPERIMENT SETUP

Figure 3.1: Structure of the test bench.

An engine-dynamometer system can make the engine under different working modes like static or varied.

Sensors are mounted on different positions of the engine. For example, in the intake manifold we concern the air temperature and the air mass flow rate, and in the exhaust manifold, we care about the air-fuel ratio of exhaust gas. Moreover, the information is needed like rotational speed, load on the engine, temperature of cooling water, and so on. Engine and dynamometer have their electronic control units as shown in Figure 3.1. A box named as engine control unit (ECU) box directly communicates with actuators in the engine. The control logic program is saved in the ECU.

Operation platform provides a user communication with the test bench. Through it we can design a control program and build it. Also, it shows real-time measured values and sends manual regulation from the user. The dSPACE hardware and software systems provide the platform for program on computer and connection with electronic controllers as shown in figure 3.1.

Control devices communicate with each other through field bus and in this test bench we use CAN bus as shown figure 3.1. Moreover, an analog-to-digital
converter (A/D) is necessary for analogue signal from sensors or their signal amplifiers. Their connection requires wires which are shielded with silver tape for preventing signal interference. The dSPACE hardware often supplies a LAN interface with computer.

### 3.2 Engine and Sensors

Figure 3.2 shows the whole engine system used in the test bench. Fresh air passes through the air filter and the intake manifold and goes into cylinders for combustion with fuel. Throttle is equipped in the air path for modifying air charge. Exhaust manifold collects burnt gas from cylinders, and then TWC converts the exhaust gas into pollution-free gas. Finally, gas goes into the surrounding through a pipe.

![Image of engine system](image)

**Figure 3.2:** The engine used in the test bench.

The air intake system is composed of an air filter, an air path pipe, a throttle, a chamber, and multiple intake manifold. Air filter is for cleaning air. The air path pipe is in a length about 5 [feet], which makes the intake port far enough from the engine body for fresh air. All cylinders share a throttle and an air chamber, and have individual intake manifolds.
3. EXPERIMENT SETUP

The engine used is a V6 engine in the test bench. In the gasoline engine, six cylinders are mounted two banks, and each bank has three cylinders. The technical data of the engine is listed in Table 3.1.

<table>
<thead>
<tr>
<th>Engine</th>
<th>Lexus 3.5 V6 engine</th>
</tr>
</thead>
<tbody>
<tr>
<td>Arrangement of the cylinders</td>
<td>V</td>
</tr>
<tr>
<td>Number of cylinders [-]</td>
<td>6</td>
</tr>
<tr>
<td>Displacement [L]</td>
<td>3.456</td>
</tr>
<tr>
<td>Manifold volume [L]</td>
<td>6.833</td>
</tr>
<tr>
<td>Fuel injection</td>
<td>D-4S</td>
</tr>
<tr>
<td>Injection sequence</td>
<td>1-2-3-4-5-6</td>
</tr>
<tr>
<td>Bore [mm]</td>
<td>94</td>
</tr>
<tr>
<td>Stroke [mm]</td>
<td>83</td>
</tr>
<tr>
<td>Combustion chamber volume [mm$^3$]</td>
<td>53.33</td>
</tr>
<tr>
<td>Compression ratio [-]</td>
<td>11.8</td>
</tr>
<tr>
<td>Maximum torque [Nm] / [rpm]</td>
<td>377 / 4800</td>
</tr>
<tr>
<td>Maximum power [kW] / [rpm]</td>
<td>227 / 6400</td>
</tr>
</tbody>
</table>

The engine is a four-stroke engine, in which the crankshaft make two revolutions during an engine cycle. During a cycle, the piston completes four separate strokes including intake, compression, combustion, and exhaust. Each cylinder has its fuel injector, air intake valve, igniter, and exhaust valve. The six cylinders work independently in a sequence. The interval between adjacent cylinders is same and 120° in crankshaft angle. Six cylinders are numbered as No. 1 cylinder to No. 6 cylinder according to the working sequence. No. 1, No. 3, and No. 5 cylinders are on a bank and other three cylinders are on the other bank.

There are two fuel injectors in an individual cylinder. One is mounted directly in the cylinder, and it is the so-called direct injection. The other one is mounted in the air intake path, and it is so-called port injection. Two fuel injectors can work independently or cooperatively. Fuel injectors can automatically modify opening time based on present fuel pressure. Two kinds of fuel injection require
3.2 Engine and Sensors

different fuel pressure. The direct injector needs much higher fuel pressure for the high in-cylinder gas pressure.

There are two exhaust systems for two banks, respectively. A photo of one exhaust system is shown in figure 3.3. The exhaust manifold collects exhaust gas from No. 1, No. 3, and No. 5 cylinders and sensors are mounted on it for measuring temperature and air-fuel ratios. The catalytic converter used is a three-way catalytic converter (TWC) in the tail pipe.

![Figure 3.3: The exhaust system used in the test bench.]

All sensors are mounted at different positions for monitoring the engine. Temperature sensors and pressure sensors are mounted in the air-intake path, and the air mass flow is gotten based on these sensor outputs. In fuel path, fuel pressure and fuel flow are concerned. In-cylinder pressure sensors are mounted for observing in-cylinder combustion, and exactly, they are combined with spark plugs. The measuring range of the in-cylinder pressure sensors is between 0 and 3626 [psi] and the sensitivity is about 0.69 [mV/psi]. Moreover, in the engine, there are sensors for rotational speed, torque, cam position, etc.

In the exhaust system, temperature sensors are mounted on the tailpipe before TWC. In this research, the air-fuel ratio of exhaust gas is focused on, and air-fuel ratio sensors are mounted in runners of individual cylinders and their exhaust manifold. All air-fuel ratio sensors used are wide-band universal exhaust gas oxygen sensors and they are with heaters. The working temperature is between 300°C and about 600°C.
3. EXPERIMENT SETUP

Moreover, cooling system and lubricating system are necessary in an entire engine system. Water cooling is used for the engine. Pumps are equipped for cooling and lubricating systems and temperature sensors are mounted for water and lubricating oil.

3.3 Dynamometer

The dynamometer used is a HORIBA Dynas3 LI dynamometer in the test bench. Through a manual transmission, the dynamometer is connected with the engine as shown in figure 3.4. The dynamometer can either drive or be driven by the engine. The two cases are often called as driving and absorbing. The dynamometer is with low moments of inertia and high overload capacity, which guarantee a highly dynamics response. Main technical data is listed in Table 3.2.

<table>
<thead>
<tr>
<th>Table 3.2: Technical data of dynamometer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dynamometer type</td>
</tr>
<tr>
<td>Rated power (absorbing) [kW]</td>
</tr>
<tr>
<td>Rated power (driving) [kW]</td>
</tr>
<tr>
<td>Rated speed (absorbing) [rpm]</td>
</tr>
<tr>
<td>Rated speed (driving) [rpm]</td>
</tr>
<tr>
<td>Rated torque (absorbing) [Nm]</td>
</tr>
<tr>
<td>Rated torque (driving) [Nm]</td>
</tr>
<tr>
<td>Moment of inertia, machine [kgm(^2)]</td>
</tr>
<tr>
<td>Max. speed gradient up to rated speed [rpm/s]</td>
</tr>
</tbody>
</table>

A controller platform which is named as SPARC is used to control the dynamometer. Actually, SPARC is combined with a control box and a power cabinet for dynamometer control. There are multiple I/O boards in SPARC for their connection. In SPARC, there is a PC architecture CPU with a ready-made operation system. A user interface is also provided in SPARC as shown in figure 3.5. Users can set operation parameters and manually control the dynamometer on the user interface.
3.3 Dynamometer

**Figure 3.4:** Dynamometer and engine.

**Figure 3.5:** User interface of SPARC.
3. EXPERIMENT SETUP

Control parameters in SPARC can also be set by external command since there are multiple interfaces like USB, RS232, CAN port, etc. In this test bench, a named AandD device is used to communicate with SPARC through CAN bus. The control project is programmed in AandD. Also, the CAN bus can be connected with engine CAN bus, then the engine and the dynamometer are controlled in same program.

3.4 Control Desk and Experiment Operation

Figure 3.6 shows the operating platform of the test bench. It mainly consists of four parts. They are monitors for monitoring the engine and the dynamometer in the inner room, meters, data processing devices, and computers for programming and showing experimental data.

![Operating platform](image)

**Figure 3.6:** Operating platform

The control structure of the test bench is shown in figure 3.7. Control devices communicate with each other through CAN bus. On the engine side, ECU sends real-time information of the engine to dSPACE and gets command sent by dSPACE. Similarly, on the dynamometer side, the control box AandD can get real-time information of dynamometer from SPARC and also controls the dynamometer by sending command to SPARC. Both sides are also connected...
3.4 Control Desk and Experiment Operation

through CAN bus, then we can build a control project on dSPACE or AandD for coordinated control of engine and dynamometer.

![Control structure of the test bench.](image)

**Figure 3.7:** Control structure of the test bench.

Before building a control program, we need to install Matlab and dSPACE software in PC and a dSPACE link board for connecting PC with the external dSPACE hardware. We can build a project in Simulink which is integrated into MATLAB as an interactive environment for modeling and analyzing. The program consists of four parts including setting, input module, program body, and output module as shown in figure 3.8. Signals are gotten from the input module and the calculated control inputs are sent into the output module. The control strategy and data calculation are involved in the program body. Simulink can automatically generate C code from Simulink block diagrams and the code is transited to the dSPACE’s hardware.

For external dSPACE hardware, we choose DS1006 processor with multiple cores and 2.6 GHz processor clock. A DS4302 CAN interface board is used which supplies four CAN channels for communication with ECU, SPARC, etc. A DS2004 high-speed A/D board is equipped for external analog sensor signals. Moreover, DS4003 provides a large number of digital I/O channels for various
3. EXPERIMENT SETUP

Figure 3.8: Program on Simulink.

applications. Through LAN bus, the external hardware is connected with the link board in PC.

The dSPACE control desk can be used to start application once the hardware is installed and the computer has booted up. On the desk, layout windows provide a graphical user interface (GUI) to the system variables. Figure 3.9 shows a constructed control desk. On it, we can set parameters and save experiment data. Similarly, we also can build a control project (on PC2), download the code into AandD, and design a GUI on the AandD control desk.

Figure 3.9: A dSPACE Control desk.

The experimental process is schemed as shown in figure 3.10, and it can be
3.4 Control Desk and Experiment Operation

divided into three parts including preparation, experiment, and finishing. Experiment steps are given as follows.

Step 1: Turn on general power switch, pumps of cooling system and lubricant system, and main devices.

Step 2: Engine warm-up. Except the engine start experiment, we need to warm the engine before experiment. The warm-up process finishes until the cooling water temperature reaches 80°C, and lubricant oil temperature reaches 70°C.

Step 3: Operation condition regulation. For an experiment, the operation condition is often regulated to a specified operation condition. The process is completed through modifying throttle position and the load supplied by the dynamometer.

Step 4: Initial values setting and mode changing. It is called as automatic mode that control variables are given by original program in ECU. While manual mode means the valve is calculated by external controller. For example, fuel injection command is sent by ECU, and the value is gotten from original program in ECU (automatic mode) or our assignment (manual mode). Before the working mode of a variable is changed from automatic mode to manual mode, some initial values should be set to avoid abrupt change in the engine.

Step 5: Control experiment. Engine works under the designed control strategy, and the experimental data is saved for future analysis.

Step 6: Experiment finish. Before engine stop, slow the engine rotational speed and reduce the load via modifying throttle angle and operating dynamometer.

In this chapter, the whole test bench is introduced and a simple experiment operation is given. The engine-dynamometer system used consists of a V6 gasoline engine and a low-inertia dynamometer. Enough sensors are equipped for monitoring the engine like oxygen sensors, in-cylinder pressure sensors, fuel pressure sensor, etc. For connection with ECU, dSPACE is used, and on its platform, we can design different control strategies. Moreover, operation steps are necessary for experiment validation, which are also introduced in this chapter.
3. EXPERIMENT SETUP

Figure 3.10: Experiment steps.

POWER ON

START:
Cooling water system;
Lubricant oil system;
Monitors, meters, etc.;
dSPACE, SPARC, etc.

ENGINE START

Warm up

Experiments on engine cold start

NO

Tw >80°C
To >70°C

YES

Engine under an operation condition

Initial values setting

Auto mode => Manual

Control Start

Manual mode => Auto

Repeat?

YES

NO

POWER OFF

Close systems and turn off devices

ENGINE STOP

Slowing speed and load

Tw: Temperature of cooling water.
To: Temperature of lubricant oil.
4

Air-Fuel Ratio Model

In an engine, an exhaust system is often shared by multiple cylinders. In the exhaust manifold, the exhaust gas is the mixture of gas from multiple cylinders, and the air-fuel ratio of exhaust gas is influenced by fuel injection into all the cylinders. This chapter aims to describe the whole physical process from fuel injection to the air-fuel ratio of exhaust gas, and then gives a modeling approach of air-fuel ratio by individual cylinder fuel injection. Both cases of direct injection and port injection are considered. Finally, experiments are conducted to validate modeling approaches.

4.1 The Case of Direct Injection

4.1.1 Physics

Recall the presentation in Chapter 2 on the structure of a multicylinder engine. The physical process is considered from the fuel injection command to the air-fuel ratio measurement in exhaust manifold. To illustrate the whole process, the combustion and the exhaust system in an engine with three cylinders is sketched as an example in Fig. 4.1. Fuel injection command is sent for individual cylinders, then injected fuel is combusted with charged air in cylinders. Burnt gas is discharged from cylinders and into their corresponding runners in a sequence, and mixes together in the exhaust manifold. Here, oxygen sensor is mounted for air-fuel ratio measurement.
4. AIR-FUEL RATIO MODEL

![Exhaust System Diagram](image)

**Figure 4.1:** The exhaust system in multicylinder engines.

Some features should be noted during the physical process. First, both discrete event and continuous-time process exist in the process. For example, fuel injection command sending is discrete-time, and gas passing through runners is continuous-time. Second, cylinders are in an operation sequence with same interval. Fuel injection is excited serially for multiply cylinders. Exhaust valves of cylinders open and close in the same sequence. Third, dynamics exists during the process like gas passing through runners and sensing air-fuel ratio. Fourth, our concerned air-fuel ratio is the one of mixed exhaust gas in exhaust manifold before into TWC, not the one of an individual cylinder.

The system input is fuel injection command and the system output is the sensed air-fuel ratio. The whole process can be divided into five parts including fuel injection, in-cylinder combustion, gas passing through runners, gas mixing in exhaust manifold, and air-fuel ratio measurement. The exhaust system of an engine with $N$ cylinders can be represented with the block diagram as shown in figure 4.2. Each block represents a physical process. For No. $i$ cylinder, $u_{fi}$ denotes the fuel injection command. Notations $m_{ai}$ and $m_{fi}$ denote the charged air mass and the injected fuel mass for combustion in No. $i$ cylinder respectively. At the exhaust valve point, gas glow and its air-fuel ratio are denoted by $\dot{m}_{ei}$ and $\eta_i$, respectively. At the mixing point, let $\dot{m}_{si}$ denote the gas flow which is from No. $i$ cylinder and the whole gas flow is denoted by $\dot{m}_{s}$. The air-fuel ratio of exhaust gas and its sensor output are denoted by $\eta$ and $\eta_s$, respectively. Therefore, the
system from $u_{fi} \ (i = 1, 2, \ldots, N)$ to $\eta_s$ is focused on with consideration of the system features.

![Figure 4.2: Schematic of the process from fuel injection to fuel-air ratio.](image)

### 4.1.2 Modeling

According to the physics introduced in Section 4.1.1, a dynamic model will be constructed that describes the behavior from the fuel injection command to the fuel-air ratio measured at the gas mixing point.

It will be shown that the system can be represented as a periodic time-varying linear system, if the sampling is conducted at BDC of each cylinder, i.e. the sampling period $T_s = 4\pi/N$ ($N$ denotes the number of cylinders), and use a unified signal $u(j)$ (the fuel injection command at the sampling index $j$) as the control input which is issued to each cylinder according to the intake phase. We will start with continuous-time domain model, and then discretize the model with the sampled data at BDC, since in-cylinder burnt gas is exhausted outside when the exhaust valve opens, usually near to BDC, and the control algorithms in ECU is also triggered by BDC time.

#### 4.1.2.1 Continuous-Time Model

Essentially, the system is a hybrid system consisting of continuous time phenomenon such as the exhaust gas flows in runners and manifold, and discrete-time event such as the mass of air charge and injected fuel per cycle of each
4. AIR-FUEL RATIO MODEL

cylinder. For the latter, the sampling-and-holding signal will be used. For the seek of simplicity, in this subsection continuous-time signal will be used for all variables.

Let \( \dot{m}_{ei}(t) \) \((i = 1, 2, \ldots, N)\) denote the gas mass flow exhausted from No. \( i \) cylinder, and \( \dot{m}_{ai}(t) \) and \( \dot{m}_{fi}(t) \) represent the masses of fresh air and fuel included in \( \dot{m}_{ei}(t) \), respectively. Then under the assumption that the components in exhaust gas are the fresh air and the fuel only, we have \( \dot{m}_{ei}(t) = \dot{m}_{ai}(t) + \dot{m}_{fi}(t) \). Furthermore, suppose that the behavior of the exhaust gas passing through the runner and reaching the gas mixing point in the manifold, where the oxygen sensor is installed, can be represented as the first-order linear system with time constant \( T_i \). Then, the gas flow observed at the gas mixing point is as follows:

\[
\dot{m}_{si}(t) = \int_0^t \frac{1}{T_i} \dot{m}_{ei}(\tau)e^{-\frac{1}{T_i}(t-\tau)} d\tau \quad (4.1)
\]
or, equivalently, the air mass flow and the fuel mass flow at the same position are given by

\[
\dot{m}_{sa}(t) = \sum_{i=1}^N \dot{m}_{si}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} \dot{m}_{ai}(\tau)e^{-\frac{1}{T_i}(t-\tau)} d\tau \quad (4.4)
\]

\[
\dot{m}_{sf}(t) = \sum_{i=1}^N \dot{m}_{si}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} \dot{m}_{fi}(\tau)e^{-\frac{1}{T_i}(t-\tau)} d\tau \quad (4.5)
\]

Therefore, the total mass flow of the fresh air and the fuel at the mixing point, \( \dot{m}_{sa}(t) \) and \( \dot{m}_{sf}(t) \), can be calculated as the following equations, respectively,

\[
\dot{m}_{sa}(t) = \sum_{i=1}^N \dot{m}_{si}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} \dot{m}_{ai}(\tau)e^{-\frac{1}{T_i}(t-\tau)} d\tau \quad (4.4)
\]

\[
\dot{m}_{sf}(t) = \sum_{i=1}^N \dot{m}_{si}(t) = \sum_{i=1}^N \int_0^t \frac{1}{T_i} \dot{m}_{fi}(\tau)e^{-\frac{1}{T_i}(t-\tau)} d\tau \quad (4.5)
\]

This means that the fuel-air ratio measured at the gas mixing point is determined by

\[
\eta(t) = \frac{\dot{m}_{sf}(t)}{\dot{m}_{sa}(t)} \quad (4.6)
\]
4.1 The Case of Direct Injection

Furthermore, let \( \eta_i(t) \) be the fuel-air ratio of the gas flow from No. \( i \) cylinder, i.e,

\[
\eta_i(t) = \frac{\dot{m}_{sfi}(t)}{\dot{m}_{sai}(t)} \quad (4.7)
\]

Combining equations (4.4)-(4.7), the mixed fuel-air ratio is expressed as

\[
\eta(t) = \sum_{i=1}^{N} \gamma_i(t) \eta_i(t) \quad (4.8)
\]

where

\[
\gamma_i(t) = \frac{\dot{m}_{sai}(t)}{\sum_{i=1}^{N} \dot{m}_{sai}(t)}
\]

Meanwhile, if we focus on the in-cylinder fuel-air ratio which is determined by the mass of air charge and injected fuel per cycle, the fuel-air ratio of each cylinder can be represented as follows. Note that the mass of air charge and the injected fuel are renewed at the corresponding BDC per cycle and held with zero-order hold till the next BDC. Hence, we introduce sampling-and-holding signals \( m_{ai}^{\text{hold}}(t) \) and \( m_{fi}^{\text{hold}}(t) \) to represent the mass of air and fuel per cycle of No. \( i \) cylinder, respectively.

For example, an image of the signal \( m_{fi}^{\text{hold}}(t) \) is sketched as in Fig. 4.3 where \( T \) is an engine cycle. Then, the in-cylinder fuel-air ratio of No. \( i \) cylinder can be calculated as \( m_{fi}^{\text{hold}}(t)/m_{ai}^{\text{hold}}(t) \). Suppose there is no cycle-to-cycle coupling and recall the assumption that the exhaust gas stays in the manifold as long as the cycle. Then, this in-cylinder fuel-air ratio of No. \( i \) cylinder is equivalent to the value given by (4.7), i.e.,

\[
\eta_i(t) = \frac{\dot{m}_{sfi}(t)}{\dot{m}_{sai}(t)} = \frac{m_{fi}^{\text{hold}}(t)}{m_{ai}^{\text{hold}}(t)} \quad (4.9)
\]

Instituting this in-cylinder fuel-air ratio into the gas mixing model (4.8) with rearranged coefficients

\[
r_i(t) = \frac{\gamma_i(t)}{m_{ai}^{\text{hold}}(t)} \quad (4.10)
\]
4. AIR-FUEL RATIO MODEL

![Diagram of sampling-and-holding description of injected fuel mass.]

Figure 4.3: Sampling-and-holding description of injected fuel mass.

we have

$$\eta(t) = \sum_{i=1}^{N} r_i(t) m_{f_{hi}}^\text{hold}(t)$$

(4.11)

Note that the fuel injection command of No. $i$ cylinder is issued at the beginning of the corresponding intake phase. For the sake of simplicity, we use the sampling-and-holding signal $u_{f_{hi}}^\text{hold}$ [mmol/cycle] which is synchronized with $m_{f_{hi}}^\text{hold}(t)$ to represent the fuel injection command. Furthermore, the actual injected fuel, in the case of direct injection, is given by

$$m_{f_{hi}}^\text{hold}(t) = u_{f_{hi}}^\text{hold}(t) + d_i$$

(4.12)

where the unknown offset $d_i$ is introduced to describe cylinder-to-cylinder imbalance caused by the perturbation in gains and external disturbance, etc. Therefore, the model (4.11) is rewritten as

$$\eta(t) = \sum_{i=1}^{N} r_i(t)(u_{f_{hi}}^\text{hold}(t) + d_i)$$

(4.13)

Let the dynamics of the fuel-air ratio sensor be represented as the first-order linear system, i.e., the dynamics of sensed fuel-air ratio $\dot{\eta}_s(t)$ is written as

$$\dot{\eta}_s(t) = \frac{1}{\tau}[-\eta_s(t) + \eta(t)]$$

(4.14)

where $\tau$ is the time constant and $\eta_s(t)$ is the sensor output.

As is well-known, the transient behavior of air-fuel sensor causes delay in measurement, the shape of exhaust manifold, and the location of the sensor result
in waste time to obtain the air-fuel ratio value. The model proposed deduced above taken these effects into account using the differential equations (4.2), (4.3), and (4.14) to represent the transient behavior.

4.1.2.2 Discrete-Time Model

Let $j$ and $k$ denote the index of cycle and BDC, and initialize the BDC of No. 1 cylinder in the first cycle ($j = 0$) as $k = 0$. Then, at No. $i$ cylinder’s BDC in $j$-th cycle, the sampled-value of a continuous-time signal $x(t)$ is

$$x(k) = x(kT_s) = x(jT + (i - 1)T_s) \quad (4.15)$$

where $T_s$ is the interval between two adjacent BDCs, i.e., $T_s = T/N$. In other words, the $k$-th sampling BDC corresponds to the BDC of No. $i$ ($i = 1, 2, \ldots, N$) cylinder in the $j$-th cycle. Moreover, if we focus on the sampling-holding signal such as the fuel injection command of No. $i$ cylinder $u_{fi}^{hold}(t)$, its sampled-data is,

$$u_{fi}^{hold}(jN+i-1) = u_{fi}^{hold}(jN+i) = \ldots = u_{fi}^{hold}((j+1)N+i-1) \quad (4.16)$$

With this fact in mind, by calculating the sampled-data at each sampling time $t = kT_s$, the discrete-time model corresponding to the gas mixing model (4.13) is obtained as follows:

When $Mod(k, N) = 0$ (at No. 1 cylinder’s BDC),

$$\eta(k) = r_1(k)[u_{f1}^{hold}(k) + d_1] + \sum_{i=2}^{N} r_i(k)[u_{fi}^{hold}(k-N+i-1)+d_i] \quad (4.17)$$

... When $Mod(k, N) = p - 1$ (at No. $p$ cylinder’s BDC),

$$\eta(k) = \sum_{i=1}^{p} r_i(k)[u_{fi}^{hold}(k-p+i) + d_i] + \sum_{i=p+1}^{N} r_i(k)[u_{fi}^{hold}(k-N-p+i)+d_i] \quad (4.18)$$

... When $Mod(k, N) = N - 1$ (at No. $N$ cylinder’s BDC),

$$\eta(k) = \sum_{i=1}^{N} r_i(k)[u_{fi}^{hold}(k+N+i)+d_i] \quad (4.19)$$
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where the sampled value \( u_{fi}^{hold}(k) \) is replaced according to

\[
\begin{align*}
    u_{fi}^{hold}(k) &= \begin{cases} 
        u_{fi}^{hold}((j-1)N+i-1) & \text{if } k < jN+i-1 \\
        u_{fi}^{hold}(k) & \text{if } k = jN+i-1 \\
        u_{fi}^{hold}(jN+i-1) & \text{if } k > jN+i-1 
    \end{cases}
\end{align*}
\]

(4.20)

As mentioned in Subsection 3.1, the exhaust gas flow of each cylinder stays in the exhaust runner and manifold not longer than one cycle. Then, if the engine is operated in static model with constant speed, the aspirated air mass per cycle is fixed and the profile of air flow in exhaust gas is periodic with period \( T \), i.e., for No. \( i \) cylinder (\( i = 1, 2, \ldots, N \)), we have

\[
\dot{m}_{sai}(t) = \dot{m}_{sai}(t + T)
\]

\[
m_{ai}^{hold}(t) = C_i
\]

(4.21)

where \( C_i \) is a constant. This leads to the coefficients \( r_i(t) \) of the model (4.13) vary periodically, i.e., \( r_i(t) = r_i(t + T) \). Thus, it is deduced as

\[
r_i(k) = \begin{cases} 
    r_i(0) & \text{Mod}(k, N) = 0 \\
    r_i(1) & \text{Mod}(k, N) = 1 \\
    \vdots & \\
    r_i(N-1) & \text{Mod}(k, N) = N-1
\end{cases}
\]

(4.22)

where \( i = 1, 2, \ldots, N \).

Moreover, introduce a unified control signal \( u_f(k) \) to denote the fuel injection command for all cylinders which will be issued to corresponding cylinder as follows,

\[
u_{fi}^{hold}(k) = u_f(k), \text{ when Mod}(k, N) = i - 1
\]

(4.23)

Then, (4.17)-(4.19) become a periodic model with the unified input signal \( u_f(k) \):

When \( \text{Mod}(k, N) = 0 \),

\[
\eta(k) = r_i(0)[u_f(k) + d_i] \\
+ \sum_{i=2}^{N} r_i(0)[u_f(k-N+i-1) + d_i]
\]

(4.24)

\[
\ldots
\]

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4.1 The Case of Direct Injection

When \( \text{Mod}(k, N) = p - 1 \),

\[
\eta(k) = \sum_{i=1}^{p} r_i (p - 1) [u_f(k - p + i) + d_i] + \sum_{i=p+1}^{N} r_i (p - 1) [u_f(k - N + p + i) + d_i]
\]

\( (4.25) \)

\[
\ldots
\]

When \( \text{Mod}(k, N) = N - 1 \),

\[
\eta(k) = \sum_{i=1}^{N} r_i (N - 1) [u_f(k - N + i) + d_i]
\]

\( (4.26) \)

With sampling time \( T_s \), the discrete-time representation of equation (4.14) is written as

\[
\eta_s(k + 1) = g \eta_s(k) + (1 - g) \eta(k)
\]

\( (4.27) \)

where \( g = 1 - T_s/\tau \).

4.1.2.3 Linear Periodic Time-Varying Model

In order to get a unified expression for equations (4.24)-(4.26), two matrices are introduced:

\[
\Gamma = \begin{pmatrix}
0 & 1 & 0 & \ldots & \ldots & 0 \\
0 & 0 & 1 & \ldots & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & 0 \\
0 & \ddots & \ddots & \ddots & \ddots & 1 \\
1 & 0 & \ldots & \ldots & \ldots & 0
\end{pmatrix}_{N \times N}
\]

\[
a_k = (1 \ 0 \ \ldots \ 0)_{1 \times N}
\]

Then, the model (4.24)-(4.26) can be represented as the unified expression

\[
\eta(k) = \sum_{i=1}^{N} (a \Gamma^k R_i) a \Gamma^{(k-i+1)} M(k)
\]

\( (4.28) \)
4. AIR-FUEL RATIO MODEL

where

\[ M(k) = \begin{pmatrix}
    u_f(k) + m(k)D_0 \\
    u_f(k-1) + m(k-1)D_0 \\
    \vdots \\
    u_f(k-N+1) + m(k-N+1)D_0
\end{pmatrix} \]

\[ D_0 = \begin{pmatrix}
    d_1 \\
    \vdots \\
    d_N
\end{pmatrix}, \quad m(k) = a\Gamma^k, \quad R_i = \begin{pmatrix}
    r_i(0) \\
    \vdots \\
    r_i(N-1)
\end{pmatrix} \]

Define the state variables as

\[
\begin{align*}
    x_1(k) &= u_f(k-1) + m(k-1)D_0 \\
    x_2(k) &= u_f(k-2) + m(k-2)D_0 \\
    \vdots \\
    x_N(k) &= u_f(k-N+1) + m(k-N+1)D_0 \\
    x_N(k) &= \eta_s(k)
\end{align*}
\]

then

\[
\begin{align*}
    x_1(k+1) &= u_f(k) + m(k)D_0 \\
    x_2(k+1) &= x_1(k) \\
    \vdots \\
    x_{N-1}(k+1) &= x_{N-2}(k) \\
    x_N(k+1) &= \eta_s(k+1)
\end{align*}
\]

From (4.28), we have

\[ \eta(k) = \sum_{i=1}^{N-1} l_i(k)x_i(k) + l_N(k)[u(k) + m(k)D_0] \]  \hspace{1cm} (4.29)

where \( l_i(k) \) \( (i = 1, 2, \ldots, N) \) represent the periodic transfer from fuel mass to fuel-air ratio.

Combining the state definition, the model (4.29) and the sensor dynamics (4.27), the state system with sensor output \( \eta_s(k) \) as system output is represented as

\[
\begin{pmatrix}
    x_1(k+1) \\
    x_2(k+1) \\
    \vdots \\
    x_N(k+1)
\end{pmatrix} = A(k) \begin{pmatrix}
    x_1(k) \\
    x_2(k) \\
    \vdots \\
    x_N(k)
\end{pmatrix} + B(k)[u(k) + m(k)D_0] \]  \hspace{1cm} (4.30)
4.1 The Case of Direct Injection

\[ y(k) = C \begin{pmatrix} x_1(k) \\ x_2(k) \\ \vdots \\ x_N(k) \end{pmatrix} \]  

(4.31)

where

\[
A(k) = \begin{pmatrix}
0 & 0 & \ldots & \ldots & 0 \\
1 & 0 & \ldots & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
0 & \ldots & 1 & 0 & 0 \\
(1-g)l_1(k) & \ldots & \ldots & (1-g)l_{N-1}(k) & g
\end{pmatrix}
\]

\[
B(k) = \begin{pmatrix}
1 \\
0 \\
\vdots \\
0 \\
(1-g)l_N(k)
\end{pmatrix}
\]

\[
C = (0 \ldots, 0, 1)
\]

Obviously, this is a linear periodic time-varying model, since \(l_i(k), \ldots, l_N(k)\) and \(m(k)\) are periodic with period \(N\).

4.1.3 Simulation

An exhaust system with three cylinders is considered in the numerical simulation. For three-cylinder exhaust system, the linear periodic time-varying model (expressed by (4.30) and (4.31)) is rewritten as

\[
\begin{pmatrix}
x_1(k+1) \\
x_2(k+1) \\
x_3(k+1)
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 0 \\
1 & 0 & 0 \\
(1-g)l_1(k) & (1-g)l_2(k) & g
\end{pmatrix} \begin{pmatrix}
x_1(k) \\
x_2(k) \\
x_3(k)
\end{pmatrix} + \begin{pmatrix}
1 \\
0 \\
(1-g)l_3(k)
\end{pmatrix} (u(k) + m(k)D_0)
\]

(4.32)

\[ y(k) = (0, 0, 1) \begin{pmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \end{pmatrix} \]

(4.33)
where the model parameters, $(1-g)l_1(k)$, $(1-g)l_2(k)$, $(1-g)l_3(k)$ and $g$, will be identified and the model will be validated by simulation in this section.

According to the physical models discussed previously, a simulator is constructed on MATLAB/SIMULINK. The program structure is as shown in figure 4.4. In the simulator, the injected fuel mass of each cylinder with individual offset (4.12), the dynamics of mass flows (4.2) and (4.3), and the mixing process (4.4), (4.5) and (4.8) are involved. The physical parameters like offsets in fuel injection path, time constants of gas passing through runners and the time constant of the sensor are set as $d_1=-0.068$[mg], $d_2=0.068$[mg], $d_3=0.136$[mg], $T_1=15$[ms], $T_2=10$[ms] $T_3=5$[ms] and $\tau=50$[ms], respectively. Moreover, white noise is added on injected fuel mass per cycle.

**Figure 4.4:** Simulator of the process from fuel injection to the oxygen sensor output

Suppose the engine is operated in a static mode with speed 1600[rpm], and the air charge mass is 99[mg/cycle], 101[mg/cycle] and 102[mg/cycle] for No. 1, No. 2 and No. 3 cylinders, respectively. Then, the BDC-scaled sampling time is $T_s=25$[ms]. Same fuel injection command is issued to three cylinders and the response of the sensor output is gotten as shown in the bottom plot of Fig. 4.5. Suppose the injector disturbance is known, then the injected fuel mass into each cylinder is shown in top three plots in Fig. 4.5. With the input-output data of
4.1 The Case of Direct Injection

Figure 4.5: Data for model identification

the simulator in Fig. 4.5, after being sampled at BDCs, parameters are identified based on least square (LS) estimation and their values list in Table 4.1.

Table 4.1: Identified parameters \((\times 10^{-3})\)

<table>
<thead>
<tr>
<th>Mod((k, 3) = r)</th>
<th>(r = 0)</th>
<th>(r = 1)</th>
<th>(r = 2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 - g l_1(k))</td>
<td>1.4124</td>
<td>2.3698</td>
<td>1.4438</td>
</tr>
<tr>
<td>(1 - g l_2(k))</td>
<td>0.4890</td>
<td>0.2893</td>
<td>0.4068</td>
</tr>
<tr>
<td>(g)</td>
<td></td>
<td>0.3440</td>
<td></td>
</tr>
<tr>
<td>(1 - g l_3(k))</td>
<td>8.2323</td>
<td>7.0710</td>
<td>7.6733</td>
</tr>
</tbody>
</table>

Validation of the identified model is demonstrated in Fig.4.6, in which the unified fuel injection command \(u(k)\) is issued to each cylinder sequentially with initial assignment \(u(0) = u_{f1}(0)\), i.e., \(u(0)\) is issued to the No. 1 cylinder. The fuel mass entering three cylinders and their unified fuel mass are shown in Fig. 4.6 [a] and [b], respectively. Under the actuation of the fuel injection, the responses of the simulator and the model with the identified parameters are shown in Fig. 4.6[c],
Figure 4.6: Simulation results
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Figure 4.7: Simulation results in detail
and a view of zoom-in is shown in Fig. 4.7 in detail. Fig. 4.6[d] provides the model error and it shows less than 0.001.

4.1.4 Identification and Experimental Validation

In this experiment, the cylinders on one side bank are considered and they are renumbered from No. 1 to No. 3 according to the spark sequence (also exhaust BDC sequence).

This is a three-cylinder exhaust system and its model is represented by (4.32) and (4.33). The model parameters like $(1 - g)l_i(k) \ (i = 1, 2, 3)$ and $g$ will be identified by experiment. After the engine warms up, the opening angle of the throttle keeps at 8.6 degrees, the rotational speed is maintained at 1600rpm by the dynamometer, and the load torque is around 80Nm. Fuel injection command of three cylinder and the corresponding sensor output are sampled at BDCs for identification. Using the least square method, model parameters are identified and listed in Table 4.2, in which the parameter $g$ is much bigger than the one in the simulation since the real sensor response is much slower.

<table>
<thead>
<tr>
<th>$Mod(k, 3) = r \</th>
<th>\ r = 0 \</th>
<th>\ r = 1 \</th>
<th>\ r = 2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$(1 - g)l_1(k)$</td>
<td>0.0767</td>
<td>0.0695</td>
<td>0.0545</td>
</tr>
<tr>
<td>$(1 - g)l_2(k)$</td>
<td>0.0428</td>
<td>0.0427</td>
<td>0.0498</td>
</tr>
<tr>
<td>$g$</td>
<td></td>
<td>932.229</td>
<td></td>
</tr>
<tr>
<td>$(1 - g)l_3(k)$</td>
<td>0.1020</td>
<td>0.1534</td>
<td>0.1581</td>
</tr>
</tbody>
</table>

The experiment is conducted for model validation by comparing the sensor output $\eta_s(k)$ and the model output $y(k)$ still under speed 1600[rpm]. Let $e(k)$ denote the model error as $e(k) = y(k) - \eta_s(k)$. For quantitative analysis, mean value of error (abbreviated to MVoE), mean absolute error (MAE), and standard
4.1 The Case of Direct Injection

deviation (SD) are defined as follows,

\[
MV_{oE} = \frac{\sum_{k=1}^{M} e(k)}{M} \\
MAE = \frac{\sum_{k=1}^{M} |e(k)|}{M} \\
SD = \frac{1}{M-1} \sqrt{\sum_{k=1}^{M} (e(k) - MV_{oE})^2}
\]

where \( M \) is the number of data points.

In the following, two experiments are conducted under open-loop and closed-loop control of air-fuel ratio, respectively. In experiment I, the fuel injection mass is adjusted in close-loop control, and Fig.4.8 provides experimental results. Fig.4.9 shows the detail of Fig.4.8 from the 280-th to the 296-th BDC, in which the model input represents fuel injection command of an individual cylinder at its corresponding BDC time as presented in modeling process.

Figure 4.8: Experimental results in Experiment I.

In Experiment II, the fuel injection mass is adjusted in open-loop control, and the comparison of the model output and the sensed fuel-air ratio is shown in Fig.4.10. During the period 530-th to 580-th BDC, fuel injection command of No. 3 cylinder is reduced to 21.5[mml/\text{stroke}] from 22.5[mml/\text{stroke}], and the detail of fuel-air ratio is shown in Fig.4.10. Analysis on model error for above experiment lists in Table 4.3.
4. AIR-FUEL RATIO MODEL

Figure 4.9: Details of Fig. 4.8

Figure 4.10: Experimental results in Experiment II.
4.2 The Case of Port Injection

### Table 4.3: Model error \((\times 10^{-4})\)

<table>
<thead>
<tr>
<th></th>
<th>MVoE</th>
<th>MAE</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Exp. I</td>
<td>0</td>
<td>1.27</td>
<td>1.59</td>
</tr>
<tr>
<td>Exp. II</td>
<td>0</td>
<td>1.33</td>
<td>1.76</td>
</tr>
</tbody>
</table>

![Graph](image)

**Figure 4.11:** Details of Experiment II.

### 4.2 The Case of Port Injection

#### 4.2.1 Physics

Fig. 4.12 provides a sketch for the engine in which three cylinders share one manifold. Fuel injection type is port injection and fuel injectors are mounted at the inlet port near to the intake valve. From the view of dynamical system, the external actuation signal is the command of fuel mass injection of each cylinder, and the output is the sensed valve from the oxygen sensor placed in the exhaust manifold.

The dynamics of the whole system from fuel injection command to the sensor
output can be explained with Fig. 4.13, in which each block represents a physical process and the denotations are defined in Section 4.2.2. First, consider the sensor dynamics in downstream represented by block (D) in Fig. 4.13. The sensor dynamics is related with the transmission characteristics, mounting location and angle of the sensor. The sensor output is the system output, i.e. the sensed fuel-air ratio $\eta_s$. The sensor input is the fuel-air ratio of the gas flow through the mixing point.

For No. $i$ cylinder, its exhaust gas passes through its runner from its exhaust valve port to the mixing point as shown in Fig. 4.12, and in Fig. 4.13, the dynamic process is represented with block (C) which is influenced by the length and the shape of its runner. In the cylinder, block (B), the fuel injected fuel is combusted with aspirated air during the combustion stroke, and the burnt gas is exhausted outside when the exhaust valve opens. We can consider that the process is triggered by impulse signals, i.e. the gas flow $\dot{m}_{ei}$ at exhaust valve port is equal to in-cylinder gas mass $m_{ei}$ multiplied by a unit impulse signal. The triggering time is at exhaust valve opening time before and near to BDC time.

In multicylinder engines, the physical process of each cylinder can also be represented with above blocks in Fig. 4.13, and the sensed fuel-air ratio is the one of the mixed gas flow. Provided N cylinders share the exhaust manifold, the

Figure 4.12: The schematic of port-injection engine system.
mixed gas flow is the sum of $\dot{m}_{si}(i = 1, 2, \ldots, N)$ where $\dot{m}_{si}$ is the gas flow from No. $i$ cylinder. However, between cylinders, fuel injection command is cited at different time and so is exhausting time. That means we can find the individual fuel injection influence factor on the fuel-air ratio in exhaust manifold, also we can modify individual fuel injection command finally for fuel-air ratio control. With the targets, in following, a fuel-air ratio model with BDC scale is modeled.

### 4.2.2 Modeling

#### 4.2.2.1 Continuous-Time Model

Essentially, the whole system is a hybrid system consisting of continuous time phenomenon such as the exhaust gas flows in runners and manifold, and discrete-time event such as sent fuel injection command, charged air mass and injected fuel per cycle of each cylinder. For the sake of simplicity, in this subsection continuous-time signal will be used for all variables. The sampling-and-holding signal will be introduced for discrete-time event.

Note that charged air mass and the injected fuel mass are renewed at the corresponding BDC per cycle and held with zero-order hold till the next BDC. Hence, we introduce sampling-and-holding signals $m_{ri}^{\text{hold}}(t)$, $m_{pi}^{\text{hold}}(t)$, $m_{fi}^{\text{hold}}(t)$,
and \( m_{\text{ai}}^{\text{hold}}(t) \) to represent injected fuel mass, fuel deposit on wall, fuel mass into cylinder No. \( i \), and charged air mass, respectively.

Following the schematic shown in Fig. 4.13, let \( u_{fi}^{\text{hold}}(t) \) and \( d_i \) be fuel injection command and fuel injector disturbance of No. \( i \) cylinder, respectively, then the injected fuel mass \( m_{ri}^{\text{hold}}(t) \) can be written as

\[
m_{ri}^{\text{hold}}(t) = u_{fi}^{\text{hold}}(t) + d_i
\] (4.34)

The wall-wetting model can be written as,

\[
\begin{align*}
m_{\text{pi}}^{\text{hold}}(t + T_c) &= (1 - \beta_i) m_{\text{pi}}^{\text{hold}}(t) + (1 - \alpha_i) m_{ri}^{\text{hold}}(t) \\
m_{fi}^{\text{hold}}(t) &= \beta_i m_{\text{pi}}^{\text{hold}}(t) + \alpha_i m_{ri}^{\text{hold}}(t)
\end{align*}
\]

where \( \alpha_i \) is the fraction of injected fuel that enters the cylinder directly each cycle and \( \beta_i \) is the fraction of fuel puddle that evaporates and enters the cylinder each cycle.

We assume that the initial time \( (t = 0) \) is at the BDC time of cylinder No. 1, then No. \( i \) cylinder’s BDC during \( j \)-th cycle occurs at \( t = jT_c + \frac{i-1}{N}T_c \) for \( N \)-cylinder system. Let \( \dot{m}_{\text{ai}}(t) \) and \( \dot{m}_{fi}(t) \) denote the air mass flow and the fuel mass flow at exhaust valve port, respectively, and they can be described with impulse signals as,

\[
\dot{m}_{\text{ai}}(t) = m_{\text{ai}}^{\text{hold}}(t) \sum_j \delta(t - jT_c - \frac{i-1}{N})
\] (4.35)

\[
\dot{m}_{fi}(t) = m_{fi}^{\text{hold}}(t) \sum_j \delta(t - jT_c - \frac{i-1}{N})
\] (4.36)

Considering the runner dynamics as described in previous section, at the mixing point in exhaust manifold, the fuel flow \( \dot{m}_{sfi}(t) \) and the air flow \( \dot{m}_{sai}(t) \) from No. \( i \) cylinder are, respectively,

\[
\dot{m}_{sai}(t) = \int_0^t \frac{1}{\tau_i} \dot{m}_{\text{ai}}(\tau) e^{-\frac{1}{\tau_i}(t-\tau)} d\tau
\] (4.37)
4.2 The Case of Port Injection

\[ \dot{m}_{sfi}(t) = \int_0^t \frac{1}{\tau_i} \dot{m}_{fi}(\tau) e^{-\frac{1}{\tau_i} (t-\tau)} d\tau \]  
(4.38)

where \( \tau_i \) represents the time constant of first-order linear dynamical system for the block (C) in Fig. 4.13. Gas from individual cylinders mixed in exhaust manifold and the fuel-air ratio \( \eta(t) \) of mixed gas in exhaust manifold is equal to

\[ \eta(t) = \frac{\sum_{i=1}^N \dot{m}_{sfi}(t)}{\sum_{i=1}^N \dot{m}_{sai}(t)} = \sum_{i=1}^N \gamma_i(t) \frac{\dot{m}_{sfi}(t)}{\dot{m}_{sai}(t)} \]  
(4.39)

with definition,

\[ \gamma_i(t) = \frac{\dot{m}_{sai}(t)}{\sum_{i=1}^N \dot{m}_{sai}(t)} \]  
(4.40)

Suppose exhaust gas stays at exhaust valve port or in exhaust manifold as long as an engine cycle, i.e. there is no cycle-to-cycle coupling, then we have

\[ \frac{\dot{m}_{sfi}(t)}{\dot{m}_{sai}(t)} = \frac{\dot{m}_{fi}(t)}{\dot{m}_{ai}(t)} = \frac{m_{fi}^{\text{hold}}(t)}{m_{ai}^{\text{hold}}(t)} \]  
(4.41)

Taking equation (4.41) into (4.39), the fuel-air ratio is rewritten as

\[ \eta(t) = \sum_{i=1}^N \gamma_i(t) \frac{m_{fi}^{\text{hold}}(t)}{m_{ai}^{\text{hold}}(t)} = \sum_{i=1}^N r_i(t) m_{fi}^{\text{hold}}(t) \]  
(4.42)

where the denotation \( r_i(t) \) is

\[ r_i(t) = \gamma_i(t) / m_{ai}^{\text{hold}}(t) \]  
(4.43)

The denotation \( \eta_s(t) \) represents the sensor output and it is

\[ \eta_s(t) = \eta(t) \odot \frac{1}{\tau} e^{-\frac{t}{\tau}} \]  
(4.44)

where \( \tau \) is the time constant of the sensor dynamics and the symbol \( \odot \) represents convolution.

Instead of unintelligible combined representation, here, we use continuous-time equations (4.34), (4.35), (4.42), and (4.44) to describe the whole system.
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4.2.2.2 Discrete-Time Representations of Model

With BDC-scale sampling, the sampling-and-holding signals like $u_{fi}^{\text{hold}}(t)$ renew at No. $i$ cylinder’s BDC and keep until its BDC in next cycle. Fig. 4.14 provides an example of $u_{fi}^{\text{hold}}(k)$ in a three-cylinder system, in which circles, squares, and sparks are used to symbolize sampled fuel injection command of No. 1, No. 2, and No. 3 cylinders, respectively. Here, we use a unified denotation $u_f^U(k)$ for $u_{fi}^{\text{hold}}(k)$ ($i = 1, 2, 3$) as shown in Fig. 4.14 where $u_f^U(k)$ is symbolized by black symbols.

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure4.14}
\caption{Unified signal for fuel injection command of all cylinders.}
\end{figure}

The unified denotation $u_f^U(k)$ can be generalized for the N-cylinder system, and it is defined as

$$u_f^U(k) = u_{fi}^{\text{hold}}(k)$$ (4.45)

for $\text{Mod}(k, N) = i - 1$, in which $\text{Mod}(k, N)$ represents the reminder when $k$ is divided by $N$. Obviously, $\text{Mod}(k, N) = i - 1$ means the $k$-th BDC is No. $i$
4.2 The Case of Port Injection

cylinder’s BDC. Thus, at No. \(i\) cylinder’s BDC, we have

\[
\begin{align*}
\hat{u}_{f1}^\text{hold}(k) &= u_{f1}^U(k - i + 1) \\
\vdots
\hat{u}_{fi}^\text{hold}(k) &= u_{f1}^U(k) \\
\vdots
\hat{u}_{fN}^\text{hold}(k) &= u_{f1}^U(k - N + i)
\end{align*}
\]

(4.46)

Similarly, unified denotations \(m_U^r(k), m_U^p(k),\) and \(m_U^f(k)\) are introduced for sampled sampling-and-holding signals \(m_{r1}^\text{hold}(k), m_{p1}^\text{hold}(k),\) and \(m_{f1}^\text{hold}(k),\) respectively.

For parameters \(d_i, \alpha_i,\) and \(\beta_i\) \((i = 1, 2, \ldots, N)\) in models (4.34) and (4.35), unified denotations \(d(k), \alpha(k),\) and \(\beta(k)\) are introduced, and they are \(d(k) = a\Gamma^kD_0, \alpha(k) = a\Gamma^k\alpha,\) and \(\beta(k) = a\Gamma^k\beta,\) respectively, in which

\[
\Gamma = \begin{pmatrix}
0 & 1 & 0 & \ldots & 0 \\
0 & 0 & 1 & \ldots & 0 \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & 0 \\
0 & \ldots & \ldots & 1 \\
1 & 0 & \ldots & \ldots & 0
\end{pmatrix}_{N \times N}
\]

(4.47)

\[a = [1, 0, \ldots, 0]_{1 \times N}, D_0 = [d_1, \ldots, d_N]^T, \alpha = [\alpha_1, \ldots, \alpha_N]^T, \beta = [\beta_1, \ldots, \beta_N]^T.\]

With above introduced unified denotations, equations (4.34), (4.35), and (4.44) are sampled at each BDC as following, respectively,

\[m_U^r(k) = u_{f1}^U(k) + d(k)\]

(4.48)

\[
\begin{align*}
m_U^r(k + 3) &= (1 - \beta(k))m_U^r(k) + (1 - \alpha(k))m_U^f(k) \\
m_U^f(k) &= \beta(k)m_U^r(k) + \alpha(k)m_U^p(k)
\end{align*}
\]

(4.49)

where \(g\) is the time constant of sensor dynamics as \(g = 1 - T_s/\tau.\) With BDC-interval sampling, the discrete-time representation of equation (4.42) is written as

\[\eta_s(k + 1) = g\eta_s(k) + (1 - g)\eta(k)\]
4. AIR-FUEL RATIO MODEL

\[ \eta(k) = [r_1(k), \ldots, r_N(k)] [m_{ai}^{\text{hold}}(k), \ldots, m_{ai}^{\text{hold}}(k)]^T = [r_1(k), \ldots, r_N(k)] [(\Gamma^k)^{-1}\Gamma^k [m_{ai}^{\text{hold}}(k), \ldots, m_{ai}^{\text{hold}}(k)]^T = [r_1(k), \ldots, r_N(k)] [(\Gamma^k)^{-1}[m_{ai}^{U}(k), \ldots, m_{ai}^{U}(k)]^T \]  

(4.50)

From equations (4.40) and (4.43), we have

\[ r_i(k) = \frac{m_{ai}^{U}(k)}{\sum_{i=1}^{N} m_{ai}^{U}(k)} \]  

(4.51)

Suppose the air mass \( m_{ai}^{\text{hold}}(k) \) \( (i = 1, \ldots, N) \) entering No. \( i \) cylinder and maps of the exhausted air flow are fixed under fixed working conditions, then the parameter \( r_i(k) \) is periodic with period \( N \). Let \( r_{ij} \) denote \( r_i(k) \) at No. \( j \) cylinder’s BDC, and we can make a table with all \( r_{ij} \) as

\[ R = \begin{pmatrix} r_{11} & \cdots & r_{1N} \\ \vdots & \ddots & \vdots \\ r_{N1} & \cdots & r_{NN} \end{pmatrix} \]  

(4.52)

Therefore, we have

\[ [r_1(k), \ldots, r_N(k)] = a\Gamma^k R \]  

(4.53)

Take (4.53) into (4.50) and \( \eta(k) \) is written as

\[ \eta(k) = a\Gamma^k R[(\Gamma^k)^{-1}[m_{ai}^{U}(k), \ldots, m_{ai}^{U}(k)]^T \]  

(4.54)

We define \( l_i(k) \) as \( [l_1(k), \ldots, l_N(k)] = a\Gamma^k R[(\Gamma^k)^{-1} \] and then (4.50) is rewritten as

\[ \eta(k) = \sum_{i=1}^{N} l_i(k)m_{ai}^{U}(k-i+1) \]  

(4.55)

Combined sensor dynamics (4.49) and equation (4.55), we have

\[ \eta_{s}(k+1) = g\eta_{s}(k)+(1-g)\sum_{i=1}^{N} l_i(k)m_{ai}^{U}(k-i+1) \]  

(4.56)

Equations (4.48), (4.49), and (4.56) are discrete-time representations of air-fuel ratio model for the N-cylinder system.
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4.2.2.3 Linear Periodic Time-Varying Model

Let the model input \( u(k) \) be the fuel injection command, i.e., \( u(k) = u_f^U(k) \), and the model output \( y(k) \) be the sensed fuel-air ratio, i.e., \( y(k) = \eta_s(k) \). States are defined as following,

\[
\begin{align*}
  x_1(k) &= m_p^U(k - 1) \\
  \vdots \\
  x_{N-1}(k) &= m_p^U(k - N + 1) \\
  x_N(k) &= m_p^U(k - N + 1) \\
  \vdots \\
  x_{2N-1}(k) &= m_p^U(k) \\
  x_{2N}(k) &= \eta_s(k)
\end{align*}
\]

(4.57)

Then, considering equations (4.48), (4.49), and (4.56), we have

\[
\begin{align*}
  x_1(k+1) &= u(k) + d(k) \\
  x_2(k+1) &= x_1(k) \\
  \vdots \\
  x_{N-1}(k+1) &= x_{N-2}(k) \\
  x_N(k+1) &= x_{N+1}(k) \\
  \vdots \\
  x_{2N-2}(k+1) &= x_{2N-1}(k) \\
  x_{2N-1}(k+1) &= \alpha(k)x_{N-1}(k) + \beta(k)x_N(k) \\
  x_{2N}(k+1) &= g x_{2N}(k) + \sum_{i=1}^{2N-1} p_i(k)x_i(k) + p_{2N}(k)[u(k) + d(k)]
\end{align*}
\]

(4.58)

where the denotations \( p_i(k) \) (\( i = 1, 2, \ldots, N \)) are

\[
\begin{pmatrix}
  p_1(k) \\
  \vdots \\
  p_{N-1}(k) \\
  p_N(k) \\
  \vdots \\
  p_{2N-1}(k) \\
  p_{2N}(k)
\end{pmatrix} = (1-g) 
\begin{pmatrix}
  l_2(k)\alpha(k-1) \\
  \vdots \\
  l_N(k)\alpha(k-N+1) \\
  l_N(k)\beta(k-N+1) \\
  \vdots \\
  l_1(k)\beta(k) \\
  l_1(k)\alpha(k)
\end{pmatrix}
\]

(4.59)

Finally, we get the state equation as

\[
\begin{align*}
  X(k+1) &= A(k)X(k) + B(k)[u(k) + d(k)] \\
  y(k) &= CX(k)
\end{align*}
\]

(4.60)
where $X(k) = [x_1(k), x_2(k), \ldots, x_{2N}(k)]^T$,

$$A(k) = \begin{pmatrix} 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ 1 & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ 0 & 1 & 0 & \ldots & \ldots & \ldots & \ldots & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & \ldots & 1 & 0 & \ldots & \ldots & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots & 0 & 0 & 1 & 0 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & \ldots & \ldots & \ldots & \alpha(k) & \beta(k) & 0 & \ldots & 0 \\ 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 & 1 & 0 \\ 0 & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & \ldots & 0 & 0 \end{pmatrix},$$

$B(k) = [1, 0, \ldots, 0, p_{2N}(k)]^T$, and $C(k) = [0, \ldots, 0, 1]$.

Recall the definition of denotations $d(k)$, $\alpha(k)$, $\beta(k)$, and $l_i(k)$ in previous subsection, they are periodic with period N. Therefore, the system 4.60 is linear periodic time-varying system.

4.2.3 Experimental Identification and Validation

4.2.3.1 Identification for Wall-Wetting Model

It is hard to directly identify the parameters $\alpha_i$ and $\beta_i$ in the wall-wetting model (4.35), since the injected fuel mass $m_{i}^{\text{hold}}(t)$ and the fuel mass entering No. $i$ cylinder $m_{fi}^{\text{hold}}(t)$ are unknown. For the sake of simplicity, injector disturbance is ignored during experimental validation. Moreover, we can get the fuel-air ratio of individual cylinder from additional oxygen sensors. When air charge is fixed, the fuel-air ratio is linearly proportional to the fuel mass $m_{fi}^{\text{hold}}(t)$. As to the time constant of the sensor itself, it can be calculated under direct-injection case.

Experience tells us that it is hard to get precise parameters through directly identifying wall-wetting model. Instead, we can find a fitting curve for the dynamical process of fuel-air ratio and then calculate wall-wetting model parameters. With continuous model, the response of fuel-air ratio can be described by

$$y(t) = 1 - \varepsilon e^{-\tau t} \quad (4.61)$$
where \( y(t) \) is the normalized fuel-air ratio. For linearization, the model is further represented as

\[
\log(1 - y(t)) = \log \varepsilon - \tau_f t
\]  

(4.62)

The following work is to find \( \log \varepsilon \) and \( \tau_f \).

Let engine work under speed 1200[rpm] and load 80[Nm]. Then, for No. 1 cylinder’s injector, the fuel injection command is changed from 21.0[mm\(^3\)/stroke] to 22.0[mm\(^3\)/stroke]. Normalize the fuel-air ratio of No. 1 cylinder \( y(t) \) as and then get \( \log(1 - y(t)) \). With identified parameters \( \log \varepsilon \) and \( \tau_f \), we can get the model parameters \( \alpha_1 = 0.2545 \) and for speed 1200[rpm] and sampling interval 0.25[ms].

Another experiment is conducted for validating parameters under rotational speed 1200[rpm]. Fuel injection command of cylinder is increased up to 22.4 [mm\(^3\)/stroke] and then reduced back to 21.4[mm\(^3\)/stroke] as shown in Fig. 4.15, in which cross symbols represent fuel injection command, squares are the estimated fuel mass into cylinder per cycle, and the red line is the corresponding fuel-air ratio. Experimental results show identified parameters and are precise. Similarly, \( \alpha_i \) and \( \beta_i \) of No. \( i \) cylinder can also be identified.

**Figure 4.15:** Experimental validation for identified wall-wetting model.
4. AIR-FUEL RATIO MODEL

4.2.3.2 Identification for the Whole Model

For three-cylinder exhaust system, the model (4.60) with \( N = 3 \) can be rewritten as

\[
\begin{aligned}
X(k + 1) &= A(k)X(k) + B(k)[u(k) + d(k)] \\ y(k) &= CX(k)
\end{aligned}
\]  

(4.63)

where

\[
X(k) = [x_1(k), \ldots, x_6(k)]^T
\]

(4.64)

\[
A(k) =
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
p_1(k) & p_2(k) & p_3(k) & p_4(k) & p_5(k) & g
\end{pmatrix}
\]

(4.65)

\[
B(k) = [1, 0, 0, 0, 0, p_6(k)]^T
\]

(4.66)

\[
C = [0, 0, 0, 0, 0, 1]^T
\]

(4.67)

With identified \( \alpha(k) \) and \( \beta(k) \) in previous subsection and the time constant \( g \), the following work is to identify the model parameter \( p_i(k) \) \((i=1, 2, \ldots, 6)\).

Equation (4.59) indicates is composed by \( l_i(k) \), \( \alpha(k) \), \( \beta(k) \), and \( g \). Therefore, we can get \( p_i(k) \) through identifying \( l_i(k) \) in equation (4.55) which is rewritten for three-cylinder system as following,

\[
\eta(k) = [l_1(k), l_2(k), l_3(k)][m_f^U(k), m_f^U(k-1), m_f^U(k-2)]^T = L(k)M_f(k)
\]

(4.68)

with

\[
L(k) = [l_1(k), l_2(k), l_3(k)]
\]

(4.69)

\[
M_f(k) = [m_f^U(k), m_f^U(k-1), m_f^U(k-2)]^T
\]

(4.70)
4.2 The Case of Port Injection

Since $L(k)$ is periodic, equation (4.68) is changed into three ones at different cylinders’ BDC and they are identified using recursive least square estimation (RLS), independently.

Following experimental operations, let engine work under the speed 1200[rpm] and the load 80[Nm]. Then, the fuel injection command is controlled under manual mode. In order for better identification, gain perturbations are added on three cylinders’ fuel injection command. Let $u_c$ note the fuel injection command, then we have three cylinders’ fuel injection as $u_{f1}^{hold} = 1.01u_c$, $u_{f3}^{hold} = 1.00u_c$, and $u_{f5}^{hold} = 0.99u_c$. Change fuel injection command and switch on the program block for online parameter identification. Fig. 4.16 provides sensed air-fuel ratio and estimated air-fuel ratio with online identified parameters. Fuel injection command of three cylinders is shown in the top plot and estimation error is given in the bottom plot in Fig. 4.16. Take the mean value of identified values as final value of $l_i(k)$, and then, with previously identified $\alpha(k)$ and $\beta(k)$, calculate parameters $p_i(k)$ ($i = 1, \ldots, 6$).

To validate the identified model, some experiments are conducted under same

![Figure 4.16: Estimation with online identified parameters.](image)
operation condition. Fuel injection control is under manual mode and gain perturbations are still added on cylinders as shown in the top plot in Fig. 4.17. In the second plot (from the top of Fig. 4.17), the estimated fuel mass entering individual cylinders each cycle is given. With identified parameters, the model output and the sensor output are shown in the third plot and their error is less than 0.1 as shown in the bottom plot in Fig. 4.17. It is noted that air-fuel ratio is used instead of fuel-air ratio in figures, since air-fuel ratio is commonly used. The mean value of estimation error is about 0.001 (accounts about 0.1% to 14.7), and the mean value of absolute error is about 0.02 (0.5% to 14.7). Fig. 4.18 provides the comparison of estimated value and sensed value when fuel injection is more than 22 [mmol /stroke]. Fig. 4.19 provides the comparison when the fuel injection command for three cylinders is same. Estimation error of air-fuel ratio in above experiments shows less than 0.1.

Figure 4.17: Experiment validation of model: case I.
4.2 The Case of Port Injection

Figure 4.18: Experiment validation of model: case II.

Figure 4.19: Experiment validation of model: case III.
4. AIR-FUEL RATIO MODEL

4.2.3.3 Variation of Model Parameters Under Different Operation Conditions

Model parameters are closely related with operation condition. For example, the time constant $g$ of oxygen sensor in the BDC-scale model is influenced by rotational speed, since the time interval between BDCs is different. Similarly, parameters $\alpha(k)$ and $\beta(k)$ in wall-wetting model are also influenced by rotational speed. In addition, the parameters $l_i(k) (i = 1, 2$ or $3)$ show the air information including air charge each cycle and air flow in exhaust gas. That means they are influenced by speed and air-charge mass (load is decided by air-charge mass). Therefore, in this part, we aim to investigate variation of model parameters under different operation conditions (speed and load).

The change of time constant $g$, and wall-wetting model parameters $\alpha(k)$ and $\beta(k)$ is just related with sampling interval. Thus, in the following, the variation of $l_i(k)$ are considered. Following the experimental operations, modify throttle until the engine works under desired operation condition. Then, change fuel control mode from automatic mode to manual mode and add perturbations on fuel injection command. Lastly, save air-fuel ratio measurement in exhaust manifold for identification using least squares. Change rotational speed and load, and above experiment is conducted again and again. Some identified parameters as representatives under different speed and load are given in Fig. 4.20, Fig. 4.21, and Fig. 4.22.

4.3 Conclusion

This chapter gives a modeling approach of air-fuel ratio for multicylinder engines following the physical process. Imbalances between cylinders in air paths, fuel paths, and exhaust runners are considered. Dynamics of the oxygen sensor is also considered.

The input is the fuel injection command of individual cylinders and the output is the sensed air-fuel ratio. A unified signal is introduced to represent individual cylinder fuel injection command at its corresponding BDC, and then the system is changed to a single-input and single-output model. The model is also linear.
4.3 Conclusion

Figure 4.20: $l_1(1)$ under different operation conditions.

Figure 4.21: $l_1(2)$ under different operation conditions.
periodic time-varying and the periodicity is due to difference cylinders. It is noted that the model is BDC-scaled and the scale is shorter than cycle scale in previous research.

The contribution of this chapter is to give a modeling approach which considers individual cylinder fuel injection. Secondly, the model is represented as a single-input and single-output model. This modeling approach provides a basis for air-fuel ratio control through individual cylinder fuel injection. Without loss of generality, the modeling approach can be used for any N-cylinder engines which have similar operation as described in this chapter.

The experiments are conducted to the modeling approach under some fixed operation conditions. Since the model parameters is different corresponding to a change of engine speed, load, etc. Experimental results show the error between measured values and the model outputs is less than $2 \times 10^{-4}$ (absolute error) which is considered as a appropriate range.

However, in practical application, it is unavoidable that operation conditions like engine speed and load change. That is a limitation on application. To solve the problem, adaptive technology or other on-line modification methods are necessary for modifying model parameters with changing operation conditions.
5

Model-Based Air-Fuel Ratio Control

Based on models given in Chapter 4, control strategies is investigated for air-fuel ratio in this chapter. First, the unknown disturbances are considered on fuel path. Learning control is used for observing disturbances, and the learnt disturbances are rejected from fuel injection command to cylinders. Second, with the disturbance compensation, predictive control technology is applied for air-fuel ratio control and limitation on inputs is considered in control design. Finally, experiments are conducted to validate above control strategies.

5.1 Disturbance Learning Control

5.1.1 Problem Statement

There is a class of control tasks, tracking under a repeatable control environment. During tracking, the target trajectory must be strictly followed from the very beginning of the execution. The repeatable control environment means an identical target trajectory and the same initialization condition for all repeated control tasks. Many existing control methods are not able to fulfill such kind of tasks, since they only warrant an asymptotic convergence, not to learn from previous control trails. Without learning, a control system can only produce the
same performance without improvement, even if the task repeats consecutively. Learning control is proposed to solve such a class of control tasks.

Learning control is not a new story. Early in 1978, Uchiyama firstly presented the idea of learning control which discussing the use of learning for high speed motion patterns of a mechanical arm [66]. In 1984, Arimoto et al. first introduced this method in English [67]. Then, a number of people began exploring the use of learning control. A main branch of learning control is the iterative learning control (ILC). The idea of ILC is straightforward, use the control information of the preceding trial to improve the control performance of the present trial [68]. This is realized through memory based on learning. A typical ILC is shown in figure 5.1, in which the system input and the system output are memorized for next control signals.

![Figure 5.1: Typical iterative learning control.](image)

Iterative learning controllers can be constructed in many different ways. In general, ILC structures can be classified into two major categories including embedded and cascaded. In the book [68], some representative ILC configurations are demonstrated, and they are most commonly used. One is called as previous cycle learning (PCL) simply because only the previous cycle control signals and previous error signals are used to form the current cycle control input. Similarly, the learning control is called as the current cycle control (CCL) when the current cycle tracking error is involved in learning. The previous and current cycle learning (PCCL) is gotten by pairing PCL and CCL together. Cascade ILC is investigated for some applications which require no re-configuration of a commercial controller. It modifies only the reference trajectory iteratively to improve the control performance.
Disturbances exist on fuel injectors and they are unknown, which is the problem in air-fuel ratio control. Solution to the problem is discussed in this section. As presented in Chapter 4, a unified disturbance is introduced to represent disturbances of individual cylinders in the modeling approach. The whole system is periodic and the (unified) disturbance is also periodic. Naturally, the iterative control is recalled for observing the unknown and periodic disturbance.

5.1.2 Learning Disturbances

The disturbance is considered as the error between fuel injection command and the real injected fuel mass. It is mainly caused by fuel pressure fluctuation in fuel path and the actuation error of a fuel injector. For a fuel injector, the disturbance can be considered as a fixed value. Figure 5.2 shows an engine system with three cylinders, in which disturbances are generated during fuel injection and they are fixed. Disturbances are denoted by \( d_1 \), \( d_2 \), and \( d_3 \). Notations \( u_1(k) \), \( u_2(k) \), and \( u_3(k) \) are fuel injection command for three cylinders, respectively. \( u(k) \) is the system input and \( y(k) \) is the system out.

![Disturbance generation](image)

**Figure 5.2:** Disturbance generation.

Fig. 5.3 shows the control structure for a three-cylinder system. Notation \( e(k) \) is the model error which is the error between the sensed fuel-air ratio and the model output. Learning control is applied to disturbance learning. The learnt disturbance is denoted by \( \hat{d}(k) \) and it represents disturbances of different cylinders at different time. Then the learnt disturbance \( \hat{d}_i(k) \) is rejected from fuel injection command to No. \( i \) cylinder (here, \( i = 1, 2, \) or 3).
Learning control technology is used to observe unknown disturbances. The disturbance is periodic and the period is assumed as $N$. Therefore, the disturbance at the past $N$-th step should be memorized and used for current disturbance. Similarly, the previous cycle error and current error are used for current disturbance learning as shown in Figure 5.4. The learning control is a previous and current cycle learning (PCCL) control according to the definition in [68].

The learning controller for disturbance learning is designed as,

$$
\hat{d}(k) = \hat{d}(k - N) + k_i e(k) + k_p(e(k) - e(k - N)) \tag{5.1}
$$

where $k_p$ and $k_i$ are the control parameters, and the model error is $e(k) = y(k) - \hat{y}(k)$. The control structure shows that it is a proportional-integral (PI) controller, and the integral interval is $N$-step, not one.
5.1 Disturbance Learning Control

Figure 5.5 shows the schematic of the learning control (5.54). The block $Z^{-N}$ represents a memory which delays input signal $N$ steps. It is just the learning block in figure 5.3.

\[ \hat{d}(k) = \hat{d}(k-3) + k_i e(k) + k_p (e(k) - e(k-3)) \]  

\[ (5.2) \]

Figure 5.5: PI controller for learning disturbances.

Above learning control can be used for on-line disturbance learning. However, the stability of the control system has not discussed yet. Experiments are conducted to validate the control strategy in following.

5.1.3 Experiment Results

To magnify the influence of disturbances of fuel paths on the air-fuel ratio, +1%, +3%, and +2% are manually added on the fuel injection command of No. 1, No. 2, and No. 3 fuel injectors, respectively.

Recall the presentation (5.54) for learning disturbance and, for $N = 3$, it can be rewritten as

\[ \hat{d}(k) = \hat{d}(k-3) + k_i e(k) + k_p (e(k) - e(k-3)) \]

Fig.5.6 provides the process of online learning disturbances from the time 11 seconds, in which the top plot shows the (same) fuel injection command issued to three fuel injectors, i.e., $u^{f_1}$, $u^{f_2}$, and $u^{f_3}$. The measured fuel-air ratio is shown in the second plot, and learnt disturbances $\hat{d}_1$, $\hat{d}_2$, and $\hat{d}_3$ are shown in the third plot.

When the shift switch in Fig.5.3 is on, learnt disturbances are removed from control inputs. The process is shown in Fig.5.7, in which the switch-on operation begins from the dashed line. After removing disturbances, fuel injection command of three cylinder are different, i.e., the control input $u(k)$ is periodic. As a
5. MODEL-BASED AIR-FUEL RATIO CONTROL

Figure 5.6: Disturbances learning.

Figure 5.7: Moving learnt disturbances from fuel injection command.
response, the air-fuel ratio rises up to about 14.4 from about 14.1. Disturbances are on learning and shown in the second plot.

Another two experiments are called as Case I and Case II, and their experimental data are shown in Figure 5.8 to Figure 5.11. In Case I, +2%, +3%, and +1% are manually added on the fuel injection command of No. 1, No. 2, and No. 3 fuel injectors, respectively. Figure 5.8 shows the on-line learnt disturbances and the process of rejecting disturbances from command is shown in Figure 5.9. In Case II, +2%, +4%, and +2% are manually added on the fuel injection command of No. 1, No. 2, and No. 3 fuel injectors, respectively. Experimental results in Case II are shown in Figure 5.10 and Figure 5.11.

5.2 Predictive Control for Air-Fuel Ratio

5.2.1 Problem Statement

In a multicylinder engine, we hope to design a control strategy for air-fuel ratio by individual cylinder fuel injection. The modeling approach for the whole process
5. MODEL-BASED AIR-FUEL RATIO CONTROL

Figure 5.9: Moving learnt disturbances from fuel injection command.

Figure 5.10: Disturbances learning.
5.2 Predictive Control for Air-Fuel Ratio

is investigated in Chapter 4 and the model input is the individual fuel injection command. Unknown disturbances in fuel paths are learnt through iterative learning control in Section 5.1. Following that, air-fuel ratio control is designed in this section.

The predictive technology is applied to air-fuel ratio control. Specifically, the control is designed based on a model which is a linear periodic time-varying system. The model is rewritten as,

\[
\begin{align*}
    X(k+1) &= A(k)X(k) + B(k)[u(k) + m(k)D_0] \\
    y(k) &= CX(k)
\end{align*}
\]  

(5.3)

where \(A(k), B(k),\) and \(m(k)\) are periodic with period \(N\) for a \(N\)-cylinder exhaust system, \(D_0\) is a fixed vector, and \(C\) is a constant matrix.

As this subsection title says, the problem is stated as the application to the linear periodic time-varying system. The predictive control strategy is designed by minimizing the cost function \(J(k)\)

\[
J(k) = \sum_{i=1}^{N_p} [\gamma - \hat{y}(k+i/k)]^2 + r_\omega \sum_{j=0}^{N_p-1} [u_0(k+j) - u(k+j)]^2
\]  

(5.4)
where the first term is linked to the predicted outputs while the second term reflects the consideration on future inputs. Denotation \( \hat{y}(k + i/k) \) is the \( i \)-step ahead prediction at \( k \)-th step.

### 5.2.2 Control Strategy

#### 5.2.2.1 Predicting Model Output

Based on the model (4.30), design a predictive control for air-fuel ratio control in this part [69, 70]. For simplicity, disturbances are not considered in this subsection, i.e., \( d = 0 \). The cost function is defined as

\[
J(k) = \sum_{i=1}^{N_p} [\gamma - \hat{y}(k+i/k)]^2 + r_\omega \sum_{j=0}^{N_c-1} [u_0(k+j) - u(k+j)]^2
\]

(5.5)

where the denotation \( \gamma \) is the objective value of the model output, \( \hat{y}(k + i/k) \) is the predicted output variable at \( k + i \), \( u_0(k) \) is the input when the system is stable, and \( u(k + j) \) is the future control movement. \( N_p \) and \( N_c \) are the length of prediction horizon and control horizon, respectively. \( r_\omega \) represents the weight of future inputs on the cost function and is used as a tuning parameter for desired closed-loop performance.

With known current states \( X(k) \) and the future control strategy \( U(k) \) which is

\[
U(k) = [ u(k) \ u(k+1) \ \ldots \ u(k+N_c-1) ]^T
\]

(5.6)

the future (\( N_p \) steps) model outputs can be written as follows,

\[
\begin{align*}
\hat{y}(k+1/k) &= CA(k)X(k) + CB(k)u(k) \\
\hat{y}(k+2/k) &= CA(k+1)A(k)X(k) + CA(k+1)B(k)u(k) \\
&\quad + CB(k+1)u(k+1) \\
&\vdots \\
\hat{y}(k+N_p/k) &= C \prod_{i=1}^{N_p} A(k + N_p - i)X(k) \\
&\quad + C \prod_{i=1}^{N_p-1} A(k + N_p - i)B(k)u(k) \\
&\quad + C \prod_{i=1}^{N_p-2} A(k + N_p - i)B(k+1)u(k+1) \\
&\quad \vdots \\
&\quad + C \prod_{i=1}^{N_p-N_c} A(k + N_p - i)B(k+N_c-1)u(k+N_c-1)
\end{align*}
\]

(5.7)
5.2 Predictive Control for Air-Fuel Ratio

The prediction function can be simplified as,

\[ Y(k) = F(k)X(k) + \Phi(k)U(k) \]  

(5.8)

where

\[ Y(k) = [ \hat{y}(k+1/k) \ \hat{y}(k+2/k) \ \ldots \ \hat{y}(k+N_p/k)]^T \]  

(5.9)

\[ F(k) = \begin{bmatrix} CA(k) \\ CA(k+1)A(k) \\ \vdots \\ C \prod_{i=1}^{N_p} A(k + N_p - i) \end{bmatrix} \]  

(5.10)

\[ \Phi(k) = \begin{bmatrix} CB(k) & 0 & \ldots & 0 \\ CA(k+1)B(k) & CB(k+1) & \ldots & 0 \\ \vdots \\ M_{N_p-1} & M_{N_p-2} & \ldots & M_{N_p-N_c-2} \end{bmatrix} \]  

(5.11)

with the denotation \( M_{N_p-i} = \prod_{i=1}^{N_p-1} A(k + N_p - i) \).

5.2.2.2 Calculating Equilibrium Point

Define a vector \( U_0(k) \) as,

\[ U_0(k) = [ u_0(k) \ u_0(k+1) \ \ldots \ u_0(k + N - 1)]^T \]  

(5.12)

In the LPTV system, the stable input \( u_0(k) \) is also periodic, i.e., \( u(k) = u(k+N) \). Therefore, \( U_0(k) \) can be written as,

\[ U_0(k) = M(k)U_0(0) \]  

(5.13)

with

\[ M(k) = \begin{pmatrix} 0 & 1 & 0 & \ldots & 0 \\ 0 & 0 & 1 & \ldots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & \ldots & \ldots & 1 \\ 1 & 0 & \ldots & \ldots & 0 \end{pmatrix}_{N \times N}^k \]  

(5.14)
Let $A_k^{k+j}$ be
\[
A_k^{k+j} = A(k)A(k+1)\ldots A(k+j)
\] (5.15)

Through $N$-step iteration, the model state $X(k)$ is written as,
\[
X(k) = A_{k-1}^{k-N}X(k-N) + A_{k-1}^{k-N+1}B(k-N)u(k-N) \\
\vdots \\
+ A_{k-1}^{k-1}B(k-2)u(k-2) \\
+ B(k-1)u(k-1)
\] (5.16)

It is noted that disturbances are not considered. When the system is stable, we have $X(k-N) = X(k)$ and the state $X(k)$ is gotten as,
\[
X(k) = [I_N - A_{k-1}^{k-N-1}]^{-1}P(k)U(k-N)
\] (5.17)

where $I_N$ is an identity matrix of size $N$ and the denotation $P(k)$ is
\[
P(k) = [A_{k-1}^{k-N+1}B(k-N)\ldots A_{k-1}^{k-1}B(k-2) B(k-1)]
\] (5.18)

When the system output $y(k)$ receives the objective value $\gamma$, equation (5.17) can be deduced as,
\[
\gamma = CX(k) = C[I_N - A_{k-1}^{k-N-1}]^{-1}P(k)U(k-N)
\] (5.19)

The equation is simplified as,
\[
\gamma = Q(k)M(k)U_0(0)
\] (5.20)

where $Q(k)$ is
\[
Q(k) = C[I_N - A_{k-1}^{k-N-1}]^{-1}P(k)
\] (5.21)

From equation (5.20), we have,
\[
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\gamma =
\begin{bmatrix}
Q(0)M(0) \\
Q(1)M(1) \\
\vdots \\
Q(N-1)M(N-1)
\end{bmatrix}
U_0(0)
\] (5.22)

Then $U_0(0)$ is calculated as
\[
U_0(0) =
\begin{bmatrix}
Q(0)M(0) \\
Q(1)M(1) \\
\vdots \\
Q(N-1)M(N-1)
\end{bmatrix}^{-1}
\begin{bmatrix}
1 \\
1 \\
\vdots \\
1
\end{bmatrix}
\gamma
\] (5.23)
5.2 Predictive Control for Air-Fuel Ratio

5.2.2.3 Control Law

Then cost function (5.5) can be rewritten as

\[ J(k) = [R_s - Y(k)]^T [R_s - Y(k)] + [U(k) - U_0(k)]^T R [U(k) - U_0(k)] \]  (5.24)

where \( R \) is a scalar matrix \( \gamma I_N \) and \( R_s = [1 \ 1 \ \ldots \ 1]^T \gamma \).

The necessary condition of the minimum \( J(k) \) is obtained as

\[ \frac{\partial J}{\partial U} = 0 \]  (5.25)

The solution is

\[
U(k) = [\Phi(k)^T \Phi(k) + R]^{-1} [\Phi(k)^T R_s - \Phi(k)^T F(k) X(k) + RU_0(k)]
\]  (5.26)

Because of the receding horizon control principle, we only take the first element of \( U(k) \) as the current control input, i.e., \( u(k) = [1 \ 0 \ \ldots \ 0] \ U(k) \). The control input \( u(k) \) is represented as

\[ u(k) = k_y(k) \gamma - k_{mpc}(k) X(k) + k_0(k) \]  (5.27)

where

\[
k_y(k) = [1 \ 0 \ \ldots \ 0] \ [\Phi(k)^T \Phi(k) + R]^{-1} \Phi(k)^T \hat{R}
\]
\[
k_{mpc}(k) = [1 \ 0 \ \ldots \ 0] \ [\Phi(k)^T \Phi(k) + R]^{-1} \Phi(k)^T F(k)
\]
\[
k_0(k) = [1 \ 0 \ \ldots \ 0] \ [\Phi(k)^T \Phi(k) + R]^{-1} RU_0(k)
\]

We have the following simple description of the above control strategy based on a model.

**Proposition:** Consider the system (4.30) with cost function (5.5). Let \( d = 0 \). The model-based \( N_p \)-step ahead predictive optimal control which minimizes (5.35), is given as follows.

\[ u(k) = k_y(k) \gamma - k_{mpc}(k) X(k) + k_0(k) \]  (5.28)

When disturbances are considered, the control strategy in (5.27) is rewritten as,

\[ u(k) = k_y(k) \gamma - k_{mpc}(k) X(k) + k_0(k) - \hat{d}(k) \]  (5.29)
5. MODEL-BASED AIR-FUEL RATIO CONTROL

Fig. 5.12 provides the schematic of the whole control system with three cylinders, i.e., $N = 3$. The denotation $e(k)$ is the model error and it is classified according to BDCs of different cylinders. The learnt disturbance $\hat{d}_i(k)$ ($i = 1, 2, \text{or } 3$) is subtracted from the corresponding fuel injection command. The switch $C$ is operated to remove learnt disturbances from the fuel injection command.

The input switches to point $A$ or $B$. The point $A$ is for the fuel injection command which is decided from the air mass flow. The point $B$ is for MPC control. $\gamma$ is the objective value of the system output.

![Diagram of Control System](image)

**Figure 5.12:** Model predictive control with learning unknown disturbance.

### 5.2.3 Experiment Results

For this validation experiment, the operation process is designed as follows. First, start engine and warm it up to water temperature over 80°C and oil temperature over 70°C. Second, set the engine under required working conditions through setting dynamometer and modifying throttle angle, for instance, speed=1200[rpm] and load=80[Nm]. Third, set fuel-injection style (port injection, direct injection, or both) and initial values for fuel injection before fuel-injection control mode is changed to manual mode from automatic mode. Lastly, modify the fuel injection
command and save data including fuel injection command, fuel-air ratio, speed, load, etc.

To magnify the influence by disturbances, perturbation is added for each fuel injector, i.e., multiply \((1 + \delta\%)\) on fuel injection command to simulate the disturbance on a fuel injector. First, the learning module begins to learn the disturbance on fuel injectors. Second, the learnt disturbance is subtracted from fuel injection command after the learning process is stable. Third, the switch selector connects to \(B\), i.e., the model predictive control works.

It is noted that the engine speed is 1200[rpm] and the throttle angle is 6° during following experiments. For the test bench, the exhaust system is shared by three cylinders. Then the model (4.30), for \(N = 3\), can be rewritten as

\[
\begin{align*}
X(k+1) &= A(k)X(k) + B(k)[u(k) + m(k)D_0] \\
y(k) &= CX(k)
\end{align*}
\]  

with
\[
m(k) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad A(k) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ a_1(k) & a_2(k) & g \end{pmatrix},
\]
\[
B(k) = \begin{pmatrix} 1 \\ b(k) \end{pmatrix}, \quad C = (0, 0, 1), \quad D_0 = (d_1, d_2, d_3)^T, \quad \text{and} \quad d(k) = m(k)D_0.
\]

Identify the model parameters and they list in Table 5.3, in which \(Mod(k, 3)\) represents the remainder when \(k\) is divided by 3. Recall the representation in subsection 5.5.4 for learning disturbance and, for \(N = 3\), it can be rewritten as

\[
\hat{d}(k) = \hat{d}(k - 3) + k_i e(k) + k_p (e(k) - e(k - 3)) \tag{5.31}
\]

<table>
<thead>
<tr>
<th>(Mod(k,3))</th>
<th>(a_1(k))</th>
<th>(a_2(k))</th>
<th>(b(k))</th>
<th>(g)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.258</td>
<td>0.164</td>
<td>0.144</td>
<td>840</td>
</tr>
<tr>
<td>1</td>
<td>0.220</td>
<td>0.161</td>
<td>0.189</td>
<td>840</td>
</tr>
<tr>
<td>2</td>
<td>0.215</td>
<td>0.156</td>
<td>0.197</td>
<td>840</td>
</tr>
</tbody>
</table>
In the predictive control, the length of the control window and the length of the prediction window are set as $N_c = 3$ and $N_p = 3$, respectively. Then the predicted model outputs are as follows.

\[
\begin{align*}
\hat{y}(k+1/k) &= CA(k)X(k) + CB(k)u(k) \\
\hat{y}(k+2/k) &= CA(k+1)A(k)X(k) + CA(k+1)B(k)u(k) + CB(k+1)u(k+1) \\
\hat{y}(k+3/k) &= CA(k+2)A(k+1)A(k)X(k) + CA(k+2)A(k+1)B(k)u(k) + CA(k+2)B(k+1)u(k+1) + CB(k+2)u(k+2)
\end{align*}
\]

(5.32)

The control law is rewritten as,

\[
u(k) = k_y(k)\gamma - k_{mpc}(k)X(k) + k_0(k) - d(k)
\]

(5.33)

The objective value $\gamma$ is set as 0.068, i.e. $\frac{1}{14.7}$, the reciprocal of the stoichiometric value. Periodic parameters $k_0(k)$, $k_{mpc}(k)$, and $k_y(k)$ list in Table.5.4.

<table>
<thead>
<tr>
<th></th>
<th>$k_y(k)$</th>
<th>$k_{mpc}(k)$</th>
<th>$k_0(k)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mod(k,3)=0</td>
<td>142.3075</td>
<td>[ 0.0457 0.0195 95.6713 ]</td>
<td>16.79</td>
</tr>
<tr>
<td>Mod(k,3)=1</td>
<td>164.3419</td>
<td>[ 0.0497 0.0192 111.5131 ]</td>
<td>16.73</td>
</tr>
<tr>
<td>Mod(k,3)=2</td>
<td>167.4869</td>
<td>[ 0.0531 0.0203 113.6641 ]</td>
<td>17.33</td>
</tr>
</tbody>
</table>

Table 5.2: Control parameters

When the input switch is connected with the point $B$ in Fig. 5.12, the predictive control starts. The control results are shown in Fig. 5.17 in which the third plot shows four lines 1-step prediction $\hat{y}(k+1/k)$, 2-step prediction $\hat{y}(k+2/k)$, 3-step prediction $\hat{y}(k+3/k)$, and the measured fuel-air ratio. The bottom plot shows that the air-fuel ratio reaches 14.7, i.e., fuel-air ratio 0.068 which is just the objective value of the control output.

For further validating the control strategy, much more experiments are conducted with same operation process and different disturbances. In case I, disturbances $-2\%$, $+4\%$, and $+1\%$ are manually added on fuel injectors of No. 1, No. 2, and No. 3 cylinders, respectively. Then learn the disturbances, remove
5.2 Predictive Control for Air-Fuel Ratio

The control result is shown in Fig. 5.14, in which the air-fuel ratio rises up to 14.7 after the controller works. In case II, disturbances −2%, +1%, and 0% are manually added on fuel injectors of three cylinders. The control results and the online learning disturbances are shown in Fig. 5.15, in which the air-fuel ratio is down to 14.7 from 14.9.

A proportional-integral controller is used for air-fuel ratio control. The controller is written as,

\[ u(k) = u(k-1) + k_1 e(k) + k_2 e(k-1) \]  

(5.34)

where the error \( e(k) \) is \( e(k) = r - y(k) \). \( y(k) \) is the sensed fuel-air ratio. \( k_1 \) and \( k_2 \) are the control parameters.

The experiment is conducted when engine speed is 1200[rpm] and throttle angle is about 6 [degrees]. In this experiment, let the objective value be \( r = 0.068 \) and the control parameters be \( k_1 = 45 \) and \( k_2 = -5 \). The control result is shown
in Fig. 5.16. The top plot provides the fuel injection command of three cylinders and the fuel-air ratio is given in the bottom plot. PI controller works from the dashed line. First, the fuel-air ratio finally converges to the objective value 0.068. Second, fuel injection command vibrates in the range 19.3[mml] to 19.5[mml]. While the fuel injection command of individual cylinders converges to fixed values in the model-based predictive control.

In the experiment, the model-based predictive control is validated on an engine with three cylinders sharing an exhaust system. The unknown disturbances in fuel paths are gotten through learning controller and the learning process is shown. The length of prediction and control is set as 3. Control results show the fuel-air ratio can be controlled to the objective value 0.068. It is noted that this experiment is conducted under fixed engine speed and fixed throttle angle. Moreover, for comparison, the PI controller is used for air-fuel ratio control, while the experiment result shows that the control input vibrates in a range due to the cylinder-to-cylinder variation.
5.2 Predictive Control for Air-Fuel Ratio

Figure 5.15: Case II: control results.

Figure 5.16: Air-fuel ratio control by PI controller.
5. MODEL-BASED AIR-FUEL RATIO CONTROL

5.3 Input-Constrained Air-Fuel Ratio Control

5.3.1 Problem Statement

This paper aims to design a control strategy for the block Controller in the control system shown in Fig. 5.12. Explicitly, based on the linear periodic time-varying model (4.30), design a predictive control by minimizing the following cost function $J(k)$ through $u(k+j)$, ($j = 0, 1, \ldots, N_c - 1$),

$$J(k) = \sum_{i=1}^{N_p} [\gamma - \hat{y}(k+i/k)]^2 + r_\omega \sum_{j=0}^{N_c-1} [u_0(k+j) - u(k+j)]^2$$  \hspace{1cm} (5.35)

subject to constraints on the amplitude and the change rate of future model inputs as follows,

$$\begin{cases} u_{\text{min}} \leq u(k+j) \leq u_{\text{max}} \\ |u(k+j) - u(k+j - N)| \leq \Delta \end{cases} \hspace{1cm} ; j = 0, 1, \ldots, N_c - 1$$  \hspace{1cm} (5.36)

In the cost function (5.35), the first term is linked to the predicted outputs while the second term reflects the consideration on future inputs. The denotation $\gamma$ is the objective value of the model output. $\hat{y}(k+i/k)$ is the $i$-step ahead prediction at time $k$, $u(k+j)$ is the control motivation at the $(k+j)$-th step, and $u_0(k+j)$ is the stable input for the $(k+j)$-th step. $r_\omega$ represents the weight of future inputs on the cost function and is used as a tuning parameter for desired closed-loop performance. $N_c$ and $N_p$ denote the length of control horizon and prediction horizon, respectively. Denotations $u_{\text{min}}, u_{\text{max}},$ and $\Delta$ are the constraint limits on the amplitude and the change rate of the control input.

5.3.2 Control Strategy

5.3.2.1 Model-Based Prediction

Let $U(k)$ denote the vector with future control strategy as,

$$U(k) = [ u(k) \hspace{0.5cm} u(k+1) \hspace{1cm} \ldots \hspace{1cm} u(k+N_c-1) ]^T$$  \hspace{1cm} (5.37)

With the future control strategy and the known current states $X(k)$, without considering disturbances and any noise, and based on the air-fuel ratio model in
Chapter 4, the predicted \( \hat{N}_p \)-step fuel-air ratios can be written as follows,

\[
\begin{align*}
\hat{y}(k+1/k) &= CA(k)X(k) + CB(k)u(k) \\
\hat{y}(k+2/k) &= CA(k+1)A(k)X(k) + CA(k+1)B(k)u(k) + CB(k+1)u(k+1) \\
& \quad \vdots \\
\hat{y}(k+N_p/k) &= CA \prod_{i=1}^{N_p} A(k + N_p - i)X(k) \\
& \quad + CB \prod_{i=1}^{N_p-1} A(k + N_p - i)B(k)u(k) \\
& \quad + CB \prod_{i=1}^{N_p-2} A(k + N_p - i)B(k+1)u(k+1) \\
& \quad \vdots \\
& \quad + CB \prod_{i=1}^{N_p-N_c} A(k + N_p - i)B(k+N_c-1)u(k+N_c-1)
\end{align*}
\] (5.38)

The prediction function can be simplified as,

\[
Y(k) = F(k)X(k) + \Phi(k)U(k)
\] (5.39)

where

\[
Y(k) = \left[ \hat{y}(k+1/k) \quad \hat{y}(k+2/k) \quad \ldots \quad \hat{y}(k+N_p/k) \right]^T
\] (5.40)

\[
F(k) = \begin{pmatrix}
CA(k) \\
CA(k+1)A(k) \\
\vdots \\
C \prod_{i=1}^{N_p} A(k + N_p - i)
\end{pmatrix}
\] (5.41)

\[
\Phi(k) = \begin{pmatrix}
CB(k) & 0 & \ldots & 0 \\
CA(k+1)B(k) & CB(k+1) & \ldots & 0 \\
\vdots \\
M_{N_p-1} & M_{N_p-2} & \ldots & M_{N_p-N_c-2}
\end{pmatrix}
\] (5.42)

with the denotation \( M_{N_p-i} = \prod_{i=1}^{N_p-1} A(k + N_p - i) \).
For the future control strategy, the limits in (5.36) are written as,

\[
\begin{align*}
  u(k) & \leq u_{\text{max}} \\
  : & \\
  u(k + N_c - 1) & \leq u_{\text{max}} \\
  -u(k) & \leq -u_{\text{min}} \\
  : & \\
  -u(k + N_c - 1) & \leq -u_{\text{min}} \\
  u(k) & \leq \Delta + u(k - N) \\
  : & \\
  u(k + N_c - 1) & \leq \Delta + u(k + N_c - 1 - N) \\
  -u(k) & \leq \Delta - u(k - N) \\
  : & \\
  -u(k + N_c - 1) & \leq \Delta - u(k + N_c - 1 - N)
\end{align*}
\]

(5.43)

Let \( M \) be a matrix as,

\[
M = \begin{pmatrix}
    E_{N_c} \\
    -E_{N_c} \\
    E_{N_c} \\
    -E_{N_c}
\end{pmatrix}
\]

(5.44)

where \( E_{N_c} \) is \( N_c \)-order unit matrix. The constraint condition \( \gamma_{up} \) is

\[
\gamma_{up} = \begin{pmatrix}
    u_{\text{max}} \\
    : \\
    u_{\text{max}} \\
    -u_{\text{min}} \\
    : \\
    -u_{\text{min}} \\
    \Delta + u(k - N) \\
    : \\
    \Delta + u(k + N_c - 1 - N) \\
    \Delta - u(k - N) \\
    : \\
    \Delta - u(k + N_c - 1 - N)
\end{pmatrix}
\]

(5.45)

For simplicity, the constraints on model inputs are,

\[
M \ast U(k) \leq \gamma_{up}
\]

(5.46)
5.3 Input-Constrained Air-Fuel Ratio Control

5.3.2.2 Numerical Solution to an Optimal Problem

The problem in subsection 2.3 can be rewritten as:

\[
\min_{U(k)} J(k) = [R_s - Y(k)]^T [R_s - Y(k)] + [U(k) - U_0(k)]^T R [U(k) - U_0(k)]
\]

subject to

\[
M U(k) \leq \gamma_{up}
\]

and the prediction function (5.39). In (5.47), \( R \) is a scalar matrix \( \gamma \omega I_N \) and the vector \( R_s = [1 \ 1 \ \ldots \ \ 1]^T_{N \times 1} \gamma \).

The primal-dual method can be used to solve the optimization with inequality constraints (5.48), and the problem is equivalent to

\[
\max_{\lambda \geq 0} \min_{U(k)} [J(U(k)) + \lambda^T (MU(k) - \gamma_{up})]
\]

where \( \lambda \) is the Lagrangian multiplier.

Let \( M^i, \gamma_{up}^i, \) and \( \lambda^i \) be the \( i \)-th row of the \( M \) matrix, the \( i \)-th element of \( \gamma_{up} \) vector, and the \( i \)-th element of Lagrangian multiplier \( \lambda \). The inequality constraints may comprise active constraints and inactive constraints. An inequality \( M^i U(k) \leq \gamma_{up}^i(k) \) is said to be active if \( M^i U(k) = \gamma_{up}^i(k) \), and inactive if \( M^i U(k) < \gamma_{up}^i(k) \). It is clear that if the active set were known, the original problem could be replaced by the corresponding problem having equality constraints only. For the inactive constraints, let the corresponding multiplier be zero.

A algorithm, called Hildreth’s Quadratic Programming Procedure (Luenberger et al., 1969; Wismer et al., 1978), can be used to search the inactive constraints set. The active constraints are gotten together and denoted by \( \lambda_{act}, M_{act} \) and \( \gamma_{act} \). Then the problem is equivalent to

\[
\max_{\lambda_{act} \geq 0} \min_{U(k)} [J(U(k)) + \lambda_{act}^T (M_{act} U(k) - \gamma_{act})]
\]

With values of \( M_{act} \) and \( \gamma_{act} \), the optimization solution is obtained as,

\[
U(k) = -E^{-1}(k) F(k) - E(k)^{-1} M_{act}^T(k) \lambda_{act}(k)
\]
5. MODEL-BASED AIR-FUEL RATIO CONTROL

where

\[ E(k) = \Phi(k)^T \Phi(k) + R \quad (5.52) \]

\[ F(k) = RU_0 + \Phi(k)^T (R_s - FX(k)) \quad (5.53) \]

The term \(-E^{-1}(k)M_{act}^T(k)\lambda_{act}(k)\) is a correction term due to the active constraints.

5.3.2.3 Control Law

Review the denotation \(d\) in equation (4.30), which represents disturbances of fuel paths, i.e., the error between the fuel injection command and the actual amount of injected fuel mass. Actually, there is no way to know exactly the actual amount of injected fuel mass each time. That means the disturbance is unknown and cannot be measured directly.

Here, we use a proportional-integral (PI) controller to learn disturbances of fuel paths with the model error. The controller is written as,

\[ \hat{d}(k) = \hat{d}(k - N) + k_i e(k) + k_p (e(k) - e(k - N)) \quad (5.54) \]

where \(k_p\) is the proportional gain, \(k_i\) is the integral gain, and the model error is \(e(k) = y(k) - \hat{y}(k)\).

Consider the disturbances in fuel paths and take the first element of future control movement \(U(k)\) in equation (5.51) as the current control input as,

\[ u(k) = k_y(k)\gamma - k_{mpc}(k)X(k) + k_0(k) - k_c(k) - \hat{d}(k) \quad (5.55) \]

where

\[ \hat{d}(k) = m(k)\hat{D}_0 \]

\[ k_y(k) = [1 \ 0 \ \ldots \ 0] [\Phi(k)^T \Phi(k) + R]^{-1} \Phi(k)^T \tilde{R} \]

\[ k_{mpc}(k) = [1 \ 0 \ \ldots \ 0] [\Phi(k)^T \Phi(k) + R]^{-1} \Phi(k)^T F(k) \]

\[ k_0(k) = [1 \ 0 \ \ldots \ 0] [\Phi(k)^T \Phi(k) + R]^{-1} RU_0(k) \]

\[ k_c(k) = [1 \ 0 \ \ldots \ 0] [\Phi(k)^T \Phi(k) + R]^{-1} M_{act}^T \lambda_{act} \]

The input switches to point A or B. The point A is for the fuel injection command which is decided from the air mass flow. The point B is for MPC control. \(\gamma\) is the objective value of the system output.
5.3 Input-Constrained Air-Fuel Ratio Control

5.3.3 Experiment Results

In this V6 engine, each exhaust manifold is shared by three cylinders, i.e., a three-cylinder system. Then, the model (4.30), for \( N = 3 \), can be written as follows,

\[
\begin{aligned}
X(k + 1) &= A(k)X(k) + B(k)[u(k) + d(k)] \\
y(k) &= CX(k)
\end{aligned}
\]  
(5.56)

with

\[
m(k) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}^k, \quad A(k) = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ a_1(k) & a_2(k) & g \end{pmatrix}, \quad B(k) = \begin{pmatrix} 1 \\ 0 \\ b(k) \end{pmatrix},
\]

\[C = (0, 0, 1).\]

Identify the model parameters and they list in Table 5.3, in which \( \text{Mod}(k, 3) \) represents the reminder when \( k \) is divided by 3.

<table>
<thead>
<tr>
<th>( \text{Mod}(k,3) )</th>
<th>( a_1(k) )</th>
<th>( a_2(k) )</th>
<th>( b(k) )</th>
<th>( g )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.258</td>
<td>0.164</td>
<td>0.144</td>
<td>840</td>
</tr>
<tr>
<td>1</td>
<td>0.220</td>
<td>0.161</td>
<td>0.189</td>
<td>840</td>
</tr>
<tr>
<td>2</td>
<td>0.215</td>
<td>0.156</td>
<td>0.197</td>
<td>840</td>
</tr>
</tbody>
</table>

Recall the representation (5.54) for learning disturbance and, for \( N = 3 \), it can be rewritten as

\[
\hat{d}(k) = \hat{d}(k - 3) + k_i e(k) + k_p (e(k) - e(k - 3))
\]  
(5.57)

In the predictive control, the length of the control window and the length of the prediction window are set as \( N_c = 3 \) and \( N_p = 3 \), respectively. Then the predicted model outputs are as follows.

\[
\begin{aligned}
\hat{y}(k + 1/k) &= CA(k)X(k) + CB(k)u(k) \\
\hat{y}(k + 2/k) &= CA(k + 1)A(k)X(k) + CA(k + 1)B(k)u(k) \\
&\quad + CB(k + 1)u(k + 1) \\
\hat{y}(k + 3/k) &= CA(k + 2)A(k + 1)A(k)X(k) + CA(k + 2)A(k + 1)B(k)u(k) \\
&\quad + CA(k + 2)B(k + 1)u(k + 1) + CB(k + 2)u(k + 2)
\end{aligned}
\]  
(5.58)
5. MODEL-BASED AIR-FUEL RATIO CONTROL

The control law is rewritten as,

\[ u(k) = k_y(k) \gamma - k_{mpc}(k)X(k) + k_0(k) - k_c(k) - \hat{d}(k) \]  \hspace{1cm} (5.59)

The objective value \( \gamma \) is set as 0.068, i.e. \( \frac{1}{14.7} \), the reciprocal of the stoichiometric value. Periodic parameters \( k_0(k), k_{mpc}(k), \) and \( k_y(k) \) list in Table 5.4.

| Table 5.4: Control parameters (Speed 1200[rpm], Load 70[Nm]) |
|-----------------|-----------------|-----------------|
| Mod(k,3)=0 | 142.3075 | [ 0.0457 0.0195 95.6713 ] | 16.79 |
| Mod(k,3)=1 | 164.3419 | [ 0.0497 0.0192 111.5131 ] | 16.73 |
| Mod(k,3)=2 | 167.4869 | [ 0.0531 0.0203 113.6641 ] | 17.33 |

For this experiment, the operation process is designed as follows. First, start engine and warm it up to water temperature over 80°C and oil over 70°C. Second, set the working mode of dynamometer as the speed mode (engine speed 1200[rpm]). Fix the throttle angle as 6° and the load is about 70[Nm]. Third, set fuel-injection style (port injection, direct injection, or both) and initial values for fuel injection before fuel-injection control mode is changed to manual mode from automatic mode. Automatic mode means the fuel injection is controlled by original algorithm in ECU, while under manual mode the fuel injection is controlled by the new control strategy. Lastly, the new controller starts with the objective value 0.068, fuel-injection control mode is changed as manual mode, and save data including fuel injection command, fuel-air ratio, speed, model prediction, etc.

To magnify the influence of disturbances of fuel paths on the fuel-air ratio, +0%, +4%, and +1% are manually added on the fuel injection command of No. 1, No. 2, and No. 3 cylinders, respectively. Use proportional-integral controller to learn disturbances and the learnt disturbances are removed from the corresponding cylinder’s fuel injection command. Set a wide range for the model input as \( u_{\text{min}} = 10, u_{\text{max}} = 30, \) and \( \Delta = 10 \). Fig. 5.17 shows the model-based predictive control with on-line learning disturbances.
To validate the control strategy (5.59), experiments are conducted for different constraints on the model input. Since the calculated $U_0$ is $[17.5, 18, 18.5]$, the upper limit and lower limit on the model input are set 17 and 20, respectively. Experiments are divided into four cases of **Case I**: $\Delta = 10$, **Case II**: $\Delta = 0.4$, **Case III**: $\Delta = 0.2$, and **Case IV**: $\Delta = 0.1$. The control results under different limits of the change rate of the model input are shown in Fig. 5.18 to Fig. 5.25.

Fig. 5.17 shows the model-based predictive control with on-line learning disturbances. Fuel injection command for three cylinders is shown in the top plot, and the second plot shows the on-line learnt disturbances $\hat{d}_1$, $\hat{d}_2$, and $\hat{d}_3$. In the bottom plot, the red line is the sensed fuel-air ratio and other lines are prediction. The controller starts from the dashed black line about 8[Second], and then fuel-air ratio keeps around the objective value 0.068 and learnt disturbances have no much change.

**Figure 5.17:** Predictive control with on-line learning disturbances

In Case I, the constraints on the model input are $17 \leq u(k) \leq 20$ and $|u(k) - u(k - 3)| \leq 10$. The control results are shown in Fig. 5.18, in which the top...
plot shows the fuel injection command of three cylinders. The central plot shows three-step ahead prediction and the sensed fuel-air ratio. The bottom plot shows the air-fuel ratio, i.e., the reciprocal of the sensed fuel-air ratio.

After the control works, the fourth constraint is active ($\lambda(4) > 0$) at the beginning as shown in Fig. 5.19. That means the lower limit constraint on the model input is active. Note that $u_f1(k)$, $u_f2(k)$, and $u_f3(k)$ in Fig. 5.19 are the control input from the controller, while the first plot of Fig. 5.18 shows the fuel injection command to three cylinders. Therefore, before controller works, they are different. $\Delta u_f1$, $\Delta u_f2$, and $\Delta u_f3$ denote the fuel-injection-command change rates of three cylinders, respectively. They are shown in the bottom plot of Fig. 5.19, and they are less than the change rate limit 10. However, even though the lower limit constraint is active, a big overshoot exists on the air-fuel ratio in Fig. 5.18.

![Figure 5.18: Case I: Control input, prediction, and control output.](image)

In Case II, constraints on the model input are set as, $17 \leq u(k) \leq 20$ and $|u(k) - u(k - 3)| \leq 0.4$. The limit on the change rate of the model input is much more strictly than Case I. Therefore, the 8-th, 9-th, 10-th, 11-th, and 12-th
constraints are active, and all of them are the constraints on the change rate of the model input. Also, the change rates of individual cylinder fuel injection command in Fig. 5.21 show that they are limited. With strictly constraints, the sensed air-fuel ratio is with much smaller overshoot as shown in Fig. 5.20.

In Case III, constraints on the model input are set as, $17 \leq u(k) \leq 20$ and $|u(k) - u(k-3)| \leq 0.2$. The control results and active constraints are shown in Fig. 5.22 and Fig. 5.23, respectively. Air-fuel ratio shows small overshoot.

In Case IV, constraints on the model input are set as, $17 \leq u(k) \leq 20$ and $|u(k) - u(k-3)| \leq 0.1$. The control results and active constraints are shown in Fig. 5.24 and Fig. 5.25, respectively. Air-fuel ratio smoothly rises to the objective value 14.7. However, the control time is much longer than previous cases.

The control strategy is validated by experiments on a V6 engine test bench. Three cylinders on a bank be considered and the system is a three-cylinder system. The predictive control results are given under different constraint limits on the change rate of the model input. Experimental results show that the control performance is much better when change rate limit is set as 0.2.
5. MODEL-BASED AIR-FUEL RATIO CONTROL

**Figure 5.20:** Case II: Control input, prediction, and control output.

**Figure 5.21:** Case II: Active constraints, control input, and change rate of input.
5.3 Input-Constrained Air-Fuel Ratio Control

Figure 5.22: Case III: Control input, prediction, and control output.

Figure 5.23: Case III: Active constraints, control input, and change rate of input
5. MODEL-BASED AIR-FUEL RATIO CONTROL

Figure 5.24: Case IV: Control input, prediction, and control output.

Figure 5.25: Case IV: Active constraints, control input, and change rate of input.
6

Conclusion

This thesis focuses on the air-fuel ratio control for exhaust emission reduction. For high performance air-fuel ratio control, an appropriate modeling approach is needed before designing control strategies, which is to describe the physical process using the mathematical tool. In air-fuel ratio control system, we want to modify fuel injection command for air-fuel ratio. Thus, the model input and model output are fuel injection command and sensed air-fuel ratio, respectively. The proposed model is should be validated through experiments and then we can design control strategies based on the model. Finally, what we concern is the air-fuel ratio control performance.

Some problem we have to face during this investigation. First, the imbalance exists between cylinders due to the difference of cylinders, air paths, fuel paths, etc. Therefore, same fuel injection command or control variables for all cylinders are unreasonable. In this investigation, individual cylinder fuel injection command is considered in modeling and control. Second, in multicylinder engine, burnt gas from multi cylinders mixes together and our concerned air-fuel ratio is just the one of the mixed gas. Therefore, the system is with multiple inputs and a single output. From the view of control theory, it is complicated for designing controllers. Moreover, in-cylinder time delay and the dynamics of fuel paths, exhaust runners, and the oxygen sensor should be considered for a precise or effective model. We intend to use the periodic description for the physical process. Then how to design control strategy based on a periodic model. That is also a problem in this investigation.
6. CONCLUSION

In this thesis, the system is considered from individual cylinder fuel injection to air-fuel ratio sensor output. Imbalance between cylinders is also considered and treated as disturbances on fuel injection. Through using a unified signal which represents fuel injection command of individual cylinders at their corresponding BDCs, we obtain a single-input and single-output model for the system. Dynamics of the oxygen sensor and difference in air paths, exhaust runners between cylinders are considered, and the periodicity is due to them. Based on the model, learning control and predictive technologies are used for air-fuel ratio control.

The modeling approach follows the physical process and finally we obtain linear periodic time-varying model of air-fuel ratio. Experiments are conducted for parameter identification and validating the precision of the modeling approach. Experiment results show the error is small between the estimated air-fuel ratio from the model and the sensed air-fuel ratio.

Control strategies are designed based the model. Learning control technology is used for unknown disturbances learning. To amplify features, some disturbances are added manually on fuel injection. The learnt disturbances are provided and they finally keep around fixed values. Predictive control technology is applied into the periodic system control. The prediction is periodic, control parameters are periodic, and equilibrium input points are periodic. Moreover, input constraints are considered and a different optimal problem is solved. Finally, experiments are conducted for air-fuel ratio control and air-fuel ratios under different cases are given. Experiment results show the air-fuel ratio can be controlled to the desired value 14.7 and the control performance is good enough to meet the demand.

In a word, this thesis investigates the physical process of air-fuel ratio generation from fuel command, uses periodic description of the system and then it is deduced as a single-input and single-output periodic model, based on the model, learning control and predictive control technologies are applied into air-fuel ratio control.

Based on the model, much more control technologies should be investigated for higher performance air-fuel ratio control. Especially, the control methods for periodic system should be discussed. Moreover, in this thesis, the working
conditions are fixed or not changed much. In future, air-fuel ratio control under changing working conditions will be considered.

For an vehicle, air-fuel ratio is not the unique index. Air-fuel ratio is closely related to torque, rational speed, etc. Therefore, in future work, the air-fuel ratio with other indices will be investigated together. Then system is changed to a multiple-inputs and multiple-outputs system, and the modeling and control design would be much more complicated.
6. CONCLUSION
References


REFERENCES


REFERENCES


REFERENCES


REFERENCES


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Appendix A

Recursive Least Squares Estimation

A.1 Problem Statement

The parameter identification problem is a problem which occurs in the estimation of models of which the equations have some unknown parameters. Consider a linear model as,

\[ y(t) = \theta^T \varphi(t) \]  \hspace{1cm} (A.1)

where the independent variable \( \varphi(t) \in R^{n \times 1} \), the dependent variable \( y(t) \in R^{m \times 1} \), and parameters \( \theta \in R^{n \times m} \). Usually, some model parameters are unknown. We need to identify them using known \( \varphi(t) \) and its corresponding \( y(t) \).

More generally, the term \( \theta \) can be used to refer to any situation where a statistical model will invariably have more than one set of parameters which generate the same distribution of observations.
A. RECURSIVE LEAST SQUARES ESTIMATION

A.2 Parameter Estimation

In the following, brief deduction of the recursive least square estimation is given. From equation (A.1), we obtain,
\[
\begin{align*}
    y^T(t) &= \varphi^T(t)\theta \\
    y^T(t - 1) &= \varphi^T(t - 1)\theta \\
    \vdots \\
    y^T(1) &= \varphi^T(1) \\
\end{align*}
\]  
(A.2)

Then the parameter \( \theta \) is gotten as,
\[
\hat{\theta}(t) = \left[ \sum_{k=1}^{t} \varphi(k)\varphi(k)^T \right]^{-1} \left[ \sum_{k=1}^{t} \varphi(k)y(k)^T \right] 
\]  
(A.3)

Denote
\[
P(t) = \left[ \sum_{k=1}^{t} \varphi(k)\varphi(k)^T \right]^{-1} 
\]  
(A.4)

Then we have,
\[
P(t)^{-1} = \sum_{k=1}^{t} \varphi(k)\varphi(k)^T = P(t-1)^{-1} + \varphi(t)\varphi(t)^T 
\]  
(A.5)

The term in equation (A.3) can be written as,
\[
\sum_{k=1}^{t-1} \varphi(k)y(k)^T = P(t-1)^T \hat{\theta}(t-1) = [P(t)^{-1} - \varphi(t)\varphi(t)^T] \hat{\theta}(t-1) 
\]  
(A.6)

Then we have iterative estimation as,
\[
\hat{\theta}(t) = \hat{\theta}(t - 1) - P(t)\varphi(t)\varphi(t)^T \hat{\theta}(t - 1) + P(t)\varphi(t)y(t)^T \\
= \hat{\theta}(t - 1) + P(t)\varphi(t)[y(t)^T - \varphi(t)^T \hat{\theta}(t - 1)] \\
= \hat{\theta}(t - 1) + K(t)\varepsilon(t) 
\]  
(A.7)

with
\[
\varepsilon(t) = y(t)^T - \varphi(t)^T \hat{\theta}(t - 1) 
\]  
(A.8)

\[
K(t) = P(t)\varphi(t) = P(t-1)\varphi(t)[I + \varphi(t)^T P(t-1)\varphi(t)]^{-1} 
\]  
(A.9)
and

$$P(t) = P(t-1) - K(t)\varphi(t)^T P(t-1)$$ \hspace{1cm} (A.10)

Often forgetting factor $\lambda$ is considered in the estimation to forget previous data. Then the terms $K(t)$ and $P(t)$ are rewritten as,

$$K(t) = P(t-1)\varphi(t)[\lambda I + \varphi(t)^T P(t-1)\varphi(t)]^{-1}$$ \hspace{1cm} (A.11)

and

$$P(t) = \frac{1}{\lambda}[P(t-1) - K(t)\varphi(t)^T P(t-1)]$$ \hspace{1cm} (A.12)

### A.3 Summary

The recursive least square estimation is introduced and its simply deduction is also given in this chapter. Since the estimation is iterative, the estimation method can be used for adaptively correcting parameters. In general, the data in the old history is not considered or not much. Therefore, recursive least square estimation with forgetting factor is widely used in practical application.
A. RECURSIVE LEAST SQUARES ESTIMATION
Appendix B

Learning Control

B.1 Motivation

Consider a control task that requires the tracking of a pre-specified reference trajectory, and the task is repeated from iteration to iteration under same conditions. Exactly, we face a class of control tasks which is tracking in a finite interval under a repeated control environment. The repeatable control environment stands for repeatability of the process under the given control task that repeats and in the presence of repeated disturbances.

Most existing control methods including adaptive or robust control, may not be suitable for such a class of tasks. Because those control methods are not able to learn from previous task execution that may succeed or fail. Without learning a control system can only produce the same performance without improvement even if the task is repeatedly executed.

A learning control is often used to solve that problem that is tracking under repeated environment. We can find many references about learning control, and it is often called as iterative learning control or simplified as ILC. The idea of learning control is straightforward and is using control information of the preceding execution to improve the present execution. This is realized through memory-based learning. In the following, we introduce iterative learning control in practical discrete-time system.
B. LEARNING CONTROL

B.2 Iterative Learning Control

Iterative learning controllers can be constructed in many different ways. In whichever way, its exclusive part is to memorize previous information. Fig. B.1 shows the structure of a learning control, in which information of previous control inputs, objective values, and control outputs are memorized for current modification. Denotation $\gamma(k)$ is the objective trajectory and $e(k - T_1)$ is the error with $T_1$ steps delay.

![Figure B.1: Structure of learning control.](image)

To illustrate the learning control design, a simple example is given. For simplicity, the plant is written as $y(k) = P^* u(k)$, MEM 2 and MEM 3 are zero-step memories, objective trajectory is fixed $\gamma$, and the compensator is proportional $k_p$. Thus, we have the current control input as,

$$u(k) = u(k - T_0) + k_p e(k) \quad (B.1)$$

where the error is,

$$e(k) = \gamma - y(k) \quad (B.2)$$

Therefore, we have,

$$e(k) = \gamma - Pu(k - T_0) - Pk_p e(k) \quad (B.3)$$

Finally, it is changed as,

$$e(k) = [I + P * k_p]^{-1} e(k - T_0) \quad (B.4)$$

All eigenvalues of the factor $||[I + P * k_p]^{-1}||$ is required to be below 1 for all frequencies within a band.
B.3 Summary

In this chapter, learning control is simply introduced. It is can be applied for control tasks under periodic or repeated control environment and it is much better tracking than other known control methods. In Section B.2, the scheme of learning control is shown and a simple example is given to illustrate the control method.

The learning control technology is used in this thesis for learning disturbances in fuel paths. The experimental results also show that it is convenient for periodic system.
B. LEARNING CONTROL
Appendix C

Model Predictive Control System

C.1 Introduction

Model predictive control, often simplified as MPC, is an advanced method of process control that has been in use in process industries. MPC uses the current plant measurements, the current dynamics state of the process, the MPC models, and the process variable targets and limits to calculate future changes in the dependent variables. From the view of control theory, MPC design is to solve an optimal problem minimizing a cost function which is often on the future control movement and the error between predicted outputs and their target values. Predicted outputs are calculated based on a MPC model. In practice, there are some constraints on some variables, and then the optimization is a constrained optimal problem. Researches on model predictive control theory and its application can be found in many literatures [71, 72], etc.

In this chapter, we will introduce the basic ideas and terms about model predictive control. Exactly, we just introduce discrete-time predictive control. In Section C.2, predictive control of single-input and single-output system is simply introduced and an example is given as application. In Section C.3, we introduce a predictive control with constraints and give examples as application.
C. MODEL PREDICTIVE CONTROL SYSTEM

C.2 Predictive Control of Single-Input and Single-Output system

C.2.1 Control Law

Here is a model of a plant as follows,

$$\begin{align*}
    x(k+1) &= Ax(k) + Bu(k) \\
y(k) &= Cx(k)
\end{align*}$$

(C.1)

where $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times 1}$, and $C \in \mathbb{R}^{1 \times n}$. The system is single-input and single-output, and it is linear time-invariant. The following work is to design a model predictive control based on the model.

The problem can be stated as,

$$\min_{U(k)} J(k) = \sum_{i=1}^{N_p} [\gamma - \hat{y}(k+i/k)]^2 + r_\omega \sum_{j=0}^{N_c-1} [u_0 - u(k+j)]^2$$

(C.2)

where $u_0$ is the equilibrium input and the vector $U(k)$ is the future movement and it includes future $N_c$ steps control inputs, i.e.,

$$U(k) = [u(k) \ u(k+1) \ \ldots \ u(k+N_c-1)]^T$$

(C.3)

The denotation $\gamma$ is the objective value of system outputs. The predicted output $\hat{y}(k+i/k)$ is the $i$-step ahead prediction from $k$-th step.

The prediction can be written as

$$y(k+i/k) = CA^i x(k) + \sum_{j=1}^{i} CA^{i-j} Bu(k+j-1)$$

(C.4)

where $u(k+j-1) = 0$ when $j > N_c$.

The necessary condition of the minimum $J(k)$ is obtained as

$$\left. \frac{\delta J}{\delta U} \right|_{U^*(k)} = 0$$

(C.5)

Because of the receding horizon control principle, we only take the first element of the solution $U^*(k)$ as the current control input. Finally the control input $u(k)$ can be written as,

$$u(k) = k_y \gamma - k_{mpc} x(k) + k_0$$

(C.6)
C.2 Predictive Control of Single-Input and Single-Output system

Equation (C.6) is the control law and it is exactly a state feedback control. The term $k_y \gamma$ is decided by the model structure and is fixed. The parameter $k_0$ is fixed and comprises of model structure and equilibrium input.

C.2.2 Example

As application, we take an example to show the model predictive control. For simplicity, let the model and the plant be same.

The system structure is same as equation (C.2.1), and model parameters are given as follows,

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0.3 & 0.5 & 0.8 \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 0 \\ 0.4 \end{bmatrix}, \quad C = [0 \ 0 \ 1]$$ (C.7)

Prediction length and length of future control movement are set as $N_p = 6$ and $N_c = 3$, respectively.

At 0-th step, Model predictive controller starts and the control result is shown in Fig. C.1.

![Figure C.1: MPC control results.](image-url)
C.3 Predictive Control with Constraints on Input Variables

C.3.1 Control Law

The model structure is the same as the model in Section C.2.1. In this part, we consider the case that constraints exist on amplitude and change rate of the input variable. Then the problem is stated as,

\[
\min_{U(k)} J(k) = \sum_{i=1}^{N_p} [\gamma - \hat{y}(k + i/k)]^2 + r_w \sum_{j=0}^{N_c-1} [u_0 - u(k + j)]^2
\]  

(C.8)

subject to inequality constraints as

\[
\begin{cases}
    u_{\min} \leq u(k + j) \leq u_{\max} \\
    |u(k + j) - u(k + j - 1)| \leq \Delta \\
    j = 0, 1, \ldots, N_c - 1
\end{cases}
\]

(C.9)

Finally, we get the numerical solution and choose the first element as the current control input which is represented as,

\[
u(k) = k_y \gamma - k_{mpc} x(k) + k_0 - k_c(k)
\]

(C.10)

Different from the control law (C.6), in (C.10), the term \(k_c(k)\) is a correction term due to active constraints.

C.3.2 Example

Let \(N_c = 3\) and the limits in (C.9) are written as,

\[
\begin{cases}
    u(k) \leq u_{\max} \\
    u(k + 1) \leq u_{\max} \\
    u(k + 2) \leq u_{\max} \\
    -u(k) \leq -u_{\min} \\
    -u(k + 1) \leq -u_{\min} \\
    -u(k + 2) \leq -u_{\min} \\
    u(k) \leq \Delta + u(k - 3) \\
    u(k + 1) \leq \Delta + u(k - 2) \\
    u(k + 2) \leq \Delta + u(k - 1) \\
    -u(k) \leq \Delta - u(k - 3) \\
    -u(k + 1) \leq \Delta - u(k - 2) \\
    -u(k + 2) \leq \Delta - u(k - 1)
\end{cases}
\]

(C.11)
C.3 Predictive Control with Constraints on Input Variables

The control result of input-constrained model predictive control is given in Fig. C.2. The controller starts when the trigger signal is up to 1. Lambda values in the second plot show that the first, the seventh, the eighth, and the ninth constraint conditions are active. At about step 30, control output reaches the desired value 14.7 as shown in the bottom plot of Fig. C.2.